FIRST PART.

GREENLEAF'S COMMON SCHOOL ARITHMETIC.

SOWER, BARNES, & CO.,

PHILADELPHIA.

INTRODUCTION

TO THE

NATIONAL ARITHMETIC,

ON THE

INDUCTIVE SYSTEM,

COMBINING THE

ANALYTIC AND SYNTHETIC METHODS;

IN WHICH THE PRINCIPLES OF THE SCIENCE ARE FULLY EXPLAINED AND ILLUSTRATED.

DESIGNED FOR COMMON SCHOOLS AND ACADEMIES.

By BENJAMIN GREENLEAF, A.M.
AUTHOR OF THE "NATIONAL ARITHMETIC," "ALGEBRA," ETC

NEW STEREOTYPE EDITION,
WITH ADDITIONS AND IMPROVEMENTS.

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OFFICE OF THE CONTROLLERS OF PUBLIC SCHOOLS,
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PHILADELPHIA, December 14, 1859.

At a Meeting of the Controllers of Public Schools, First District of Pennsylvania, held at the CONTROLLERS' CHAMBER, on Tuesday, December 13th, 1859, the following Resolution was adopted:—

Resolved: That GREENLEAF'S COMMON SCHOOL AND NATIONAL ARITHMETICS be introduced to be used in the Public Schools of this District.

ROBERT J. HEMPHILL, *Secretary.*

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☞ Two editions of the NATIONAL ARITHMETIC, and also of the COMMON SCHOOL ARITHMETIC, one containing the ANSWERS to the examples, and the other without them, are published. Teachers are requested to state in their orders *which edition* they prefer.

PREFACE.

THE present edition of this work has been thoroughly revised and re-written, and also improved by the addition of much valuable new material, rendering it a sufficiently complete practical treatise for the majority of learners.

The arrangement is strictly progressive; the aim having been to introduce subjects in an order most in accordance with the laws governing the proper development of mind. The rules have generally been deduced from the analysis of one or more questions, so that the reasons for the methods of solution adopted are rendered intelligible to the pupil; no knowledge of a principle being required, that has not been previously illustrated and explained. In this respect, it is believed the work will be found to differ from most other arithmetics.

In preparation of the rules, definitions, and illustrations, the utmost care has been taken to express them in language simple, precise, and accurate.

The examples are of a practical character, and adapted not only to fix in the mind the principles, which they involve, but also to interest the pupil, exercise his ingenuity, and inspire a love for mathematical science.

The reasons for the operations are explained, and an attempt is made to secure to the learner a knowledge of the philosophy of the subject, and prevent the too prevalent practice of merely performing, mechanically, operations, which he does not understand.

Analysis has been made a prominent subject, and employed in the solution of questions under most of the rules, in which it could be used with any practical advantage; and it cannot be too strongly recommended to the pupil to make use of this mode of operation, where it is recommended by the author.

All the most important methods of abridging operations, applicable to business transactions, have been given a place in the work, and, so introduced, as not to be regarded as mere blind mechanical expedients, but as rational labor-saving processes.

Old rules and distinctions, which modern improvements have rendered unnecessary, and which, deservedly, are becoming obsolete, have been avoided.

Rules for finding the greatest common divisor of fractions, and for finding the least common multiple of fractions; methods of equating accounts; division of duodecimals; exchange, foreign and inland; and several important tables, are among the new features of this edition, which will be found, it is believed, very valuable.

The articles on money, weights, measures, interest, and duties, are the results of extensive correspondence and much laborious research, and are strictly conformable to present usage, and recent legislation, state and national.

Questions have been inserted at the bottom of each page, designed to direct the attention of teachers and pupils to the most important principles of the science, and fix them in the mind. It is not intended, however, nor is it desirable, that the teacher should servilely confine himself to these questions; but vary their form, and extend them at pleasure, and invariably require the pupil thoroughly to understand the subject, and give the reasons for the various steps in the operation, by which he arrives at any result in the solution of a question.

The object of studying mathematics is not only to acquire a knowledge of the subject, but also to secure mental discipline, to induce a habit of close and patient thought, and of persevering and thorough investigation. For the attainment of this object, the examples for the exercise of the pupil are numerous, and variously diversified, and so constructed as necessarily to require careful thought and reflection for the right application of principles.

The author would respectfully suggest to teachers, who may use this book, to require their pupils to become familiar with each rule before they proceed to a new one; and, for this purpose, a frequent review of rules and principles will be of service, and will greatly facilitate their progress. If the pupil has not a clear idea of the principles involved in the solution of questions, he will find but little pleasure in the study of the science; for no scholar can be pleased with what he does not understand.

BENJAMIN GREENLEAF.

BRADFORD, MASS., *August 1st*, 1856.

NOTICE.

☞ This book embraces the elementary part of the author's "COMMON SCHOOL ARITHMETIC"; and is published separately, with a view of saving expense, for such schools as may not require a more complete treatise.

CONTENTS.

ARITHMETIC.

ARTICLE 1. QUANTITY is anything that can be measured.

A *unit* is a single thing, or one.

A *number* is either a unit or a collection of units.

An *abstract number* is a number, whose units have no reference to any particular thing or quantity; as two, five, seven.

A *concrete number* is a number, whose units have reference to some particular thing or quantity; as two books, five feet, seven gallons.

ARITHMETIC is the science of numbers, and the art of computing by them.

A *rule* of arithmetic is a direction for performing an operation with numbers.

The *introductory* and *principal rules* of arithmetic are Notation and Numeration, Addition, Subtraction, Multiplication, and Division.

The last four are called the *fundamental rules*, because upon them depend all other arithmetical processes.

§ I. NOTATION AND NUMERATION.

NOTATION.

ART. 2. NOTATION is the art of expressing numbers by figures or other symbols.

There are two methods of notation in common use; the *Roman* and the *Arabic*.

QUESTIONS. — Art. 1. What is quantity? What is a unit? What is a number? What is an abstract number? What is a concrete number? What is arithmetic? What is a rule? Which are the introductory rules? What are the last four called? — Art. 2. What is notation? How many kinds of notation in common use? What are they?

Art. 3. The Roman notation, so called from its originating with the ancient Romans, employs in expressing numbers *seven* capital letters, viz.: I for *one;* V for *five;* X for *ten;* L for *fifty;* C for *one hundred;* D for *five hundred;* M for *one thousand.*

All the other numbers are expressed by the use of these letters, either in repetitions or combinations; as, II expresses *two;* IV, *four;* VI, *six,* &c.

By a *repetition* of a letter, the value denoted by the letter is represented as repeated; as, XX represents *twenty;* CCC, *three hundred.*

By writing a letter denoting a less value *before* a letter denoting a greater, their *difference* of value is represented; as, IV represents *four;* XL, *forty.* By writing a letter denoting a less value *after* a letter denoting a greater, their *sum* is represented; as, VI represents *six;* XV, *fifteen.*

A dash (—) placed over a letter increases the value denoted by the letter a *thousand* times; as, $\overline{\text{V}}$ represents *five thousand;* $\overline{\text{IV}}$, *four thousand.*

Table of Roman Letters.

I	one.	LXXX	eighty.
II	two.	XC	ninety.
III	three.	C	one hundred.
IV	four.	CC	two hundred.
V	five.	CCC	three hundred.
VI	six.	CCCC	four hundred.
VII	seven.	D	five hundred.
VIII	eight.	DC	six hundred.
IX	nine.	DCC	seven hundred.
X	ten.	DCCC	eight hundred.
XX	twenty.	DCCCC	nine hundred.
XXX	thirty.	M	one thousand.
XL	forty.	MD	fifteen hundred.
L	fifty.	MM	two thousand.
LX	sixty.	$\overline{\text{X}}$	ten thousand.
LXX	seventy.	$\overline{\text{M}}$	one million.

Questions. — Art. 3. Why is the Roman notation so called? By what are numbers expressed in the Roman notation? What effect has the repetition of a letter? What is the effect of writing a letter expressing a less value before a letter denoting a greater? What of writing the letter after another denoting a greater value? How many times is the value denoted by a letter increased by placing a dash over it? Repeat the table.

The Roman notation is now but little used, except in numbering sections, chapters, and other divisions of books.

EXERCISES IN ROMAN NOTATION.

The learner may write the following numbers in letters:

1. Ninety-six.
2. Eighty-seven.
3. One hundred and ten.
4. One hundred and sixty-nine.
5. Two hundred and seventy-five.
6. Five hundred and forty-two.
7. One thousand three hundred and nineteen.
8. One thousand eight hundred and fifty-eight.

ART. 4. The Arabic notation, so called from its having been made known through the Arabs, employs in expressing numbers *ten* characters or figures, viz.:

1,	2,	3,	4,	5,	6,	7,	8,	9,	0.
one,	two,	three,	four,	five,	six,	seven,	eight,	nine,	cipher.

The first *nine* are sometimes called *digits*, from *digitus*, the Latin signifying a finger, because of the use formerly made of the fingers in reckoning. The *cipher*, also, has sometimes been called *naught*, or *zero*, from its expressing the *absence* of a number, or *nothing*, when standing alone.

ART. 5. The particular position a figure occupies with regard to other figures is called its PLACE; as in 32, counting from the right, the 2 occupies the first place, and the 3 the second place, and so on for any other like arrangement of figures.

The digits have been denominated *significant figures*, because each expresses of itself a positive value, always representing so many *units*, or *ones*, as its name indicates. But the *size* or *value* of the units represented by a figure differs with the place occupied by the figure.

For example, there are written together to represent a number three figures, thus, 366 (three hundred and sixty-six). Each of the figures, without regard to its place, expresses units, or ones; but these units, or ones, differ in value. The 6 occupying the first place represents 6 single units; the 6 occupying the second place repre-

QUESTIONS. — What use is now made of Roman notation? — Art. 4. How many characters are employed in the Arabic notation? What are the first nine called, and why? What is the cipher sometimes called? What does it represent when standing alone? — Art. 5. What is meant by the place of a figure? What have the digits been denominated? Why? How does the size or value of units represented by figures differ?

sents 6 tens, or 6 units each *ten times* the size or value of a unit of the first place; and the 3 occupying the third place represents 3 hundreds, or 3 units each *one hundred times* the size or value of a unit of the first place.

ART. 6. The *cipher* becomes significant when connected with other figures, by filling a place that otherwise would be vacant; as in 10 (ten), where it occupies the vacant place of units; in 120 (one hundred and twenty), where it also occupies the vacant place of units; and in 304 (three hundred and four), where it fills the vacant place of tens.

ART. 7. The *simple* value of a unit is the value expressed by a figure standing alone; or, in a collection, when standing in the right-hand place. Thus 6 alone, or in 26, expresses a simple value of six single units, or ones.

The *local* value of a unit is the value expressed by a figure when it is used in combination with another figure or figures, and depends upon the place the figure occupies.

The local values expressed by figures will be made plain by the following table and its explanation.

Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.	The figures in this table are read thus:
						9	Nine.
					9	8	Ninety-eight.
				9	8	7	Nine hundred eighty-seven.
			9	8	7	6	Nine thousand eight hundred seventy-six.
		9	8	7	6	5	Ninety-eight thousand seven hundred sixty-five.
	9	8	7	6	5	4	Nine hundred eighty-seven thousand six hundred fifty-four.
9	8	7	6	5	4	3	Nine millions eight hundred seventy-six thousand five hundred forty-three.

QUESTIONS. — Art. 6. When does a cipher become significant? — Art. 7. What is the simple value of a unit? What is the local value of a unit? What is the design of this table?

It will be noticed in the preceding table, that each figure in the right-hand or units' place expresses the local value of so many *units;* but when standing in the second place, it expresses the local value of so many *tens*, each of the value of ten ones; when in the third place, the local value of so many *hundreds*, each of the value of ten tens; when in the fourth place, the local value of so many *thousands*, each of the value of ten hundreds; *the value expressed by any figure being always made tenfold by each removal of it one place to the left hand.*

NUMERATION.

ART. **8.** NUMERATION is the art of reading numbers when expressed by figures.

ART. **9.** There are two methods of numeration in common use: the *French* and the *English.*

ART. **10.** The French method is that in general use on the continent of Europe and in the United States. It separates figures into groups, called *periods*, of three places each, and gives a distinct name to each period.

FRENCH NUMERATION TABLE.

Hundreds of Sextillions. Tens of Sextillions. Sextillions.	Hundreds of Quintillions. Tens of Quintillions. Quintillions.	Hundreds of Quadrillions. Tens of Quadrillions. Quadrillions.	Hundreds of Trillions. Tens of Trillions. Trillions.	Hundreds of Billions. Tens of Billions. Billions.	Hundreds of Millions. Tens of Millions. Millions.	Hundreds of Thousands. Tens of Thousands. Thousands.	Hundreds. Tens. Units.
1 2 7,	8 9 4,	2 3 7,	8 6 7,	1 2 3,	6 7 8,	4 7 8,	6 3 8.
Period of Sextillions.	Period of Quintillions.	Period of Quadrillions.	Period of Trillions.	Period of Billions.	Period of Millions.	Period of Thousands.	Period of Units.

QUESTIONS. — Art. 7. What value is expressed by a figure standing in the right-hand or units' place? What in the second place? What in the third? How do figures increase from the right towards the left? — Art. 8. What is numeration? What are the two methods of numeration in common use? Where is the French method more generally used? Repeat the French Numeration Table. What are the names of the different periods in the table? What is the value of the numbers represented in the table expressed in words?

The value of the numbers represented in this table, expressed in words, is, One hundred twenty-seven sextillions, eight hundred ninety-four quintillions, two hundred thirty-seven quadrillions, eight hundred sixty-seven trillions, one hundred twenty-three billions, six hundred seventy-eight millions, four hundred seventy-eight thousand, six hundred thirty-eight.

The names of the periods above Sextillions, in their order, are, Septillions, Octillions, Nonillions, Decillions, Undecillions, Duodecillions, Tredecillions, Quatuordecillions, Quindecillions, Sexdecillions, Septendecillions, Octodecillions, Novemdecillions, Vigintillions, &c.

ART. **11.** The successive places occupied by figures are often called *orders*. Hence, a figure in the right-hand or units' place is called a figure of the *first* order, or of the order of *units;* a figure in the second place is a figure of the *second* order, or of the order of *tens;* in the third place, of the order of *hundreds*, and so on. Thus, in the number 1847, the 7 is of the order of *units*, 4 of the order of *tens*, 8 of the order of *hundreds*, and 1 of the order of *thousands*, each figure expressing as many units as its name indicates of that order to which it belongs; so that we read the whole number, *one thousand eight hundred and forty-seven.*

ART. **12.** From the preceding table and explanation, we deduce the following rule for numerating and reading numbers expressed by figures according to the French method.

RULE. — *Begin at the right hand, and point off the figures into periods of* THREE *places each.*

Then, commencing at the left hand, read the figures of each period, adding the name of each period excepting that of units.

EXERCISES IN FRENCH NUMERATION.

The learner may read orally, or write in words, the numbers represented by the following figures:

1.	152	5.	2254	9.	84093	13.	610711
2.	276	6.	4384	10.	98612	14.	3031671
3.	998	7.	7932	11.	592614	15.	4869021
4.	1057	8.	42198	12.	400619	16.	637313789

QUESTIONS. — Art. 10. What are the names of the periods above sextillions? — Art. 11. What are the successive places of the figures in the table called? Of what order is the first or right-hand figure? The second? The third? &c. — Art. 12. What is the rule for *numerating* and *reading* numbers according to the French method?

17.	39461928	24.	3761700137706717
18.	427143271	25.	242173562357421
19.	6301706716	26.	870037637471078635
20.	143776700333	27.	8216243812706381
21.	20463162486135	28.	2403172914376931
22.	63821024711802	29.	3761706137706167138
23.	44770630147671	30.	6101676378964306077616O7

ART. 13. To write numbers by figures according to the French method, we have the following

RULE. — *Begin at the left hand, and write in each successive order the figure belonging to it.*

If any intervening order would otherwise be vacant, fill the place by a cipher.

EXERCISES IN FRENCH NOTATION AND NUMERATION.

The learner may represent by figures, and read, the following numbers:

1. Forty-seven.
2. Three hundred fifty-nine.
3. Six thousand five hundred seventy-five.
4. Nine hundred and eight.
5. Nineteen thousand.
6. Fifteen hundred and four.
7. Twenty-seven millions five hundred.
8. Ninety-nine thousand ninety-nine.
9. Forty-two millions two thousand and five.
10. Four hundred eight thousand ninety-six.
11. Five thousand four hundred and two.
12. Nine hundred seven millions eight hundred five thousand and seventy-four.
13. Three hundred forty-seven thousand nine hundred and fifteen.
14. Eighty-nine thousand forty-seven.
15. Fifty-one thousand eighty-one.
16. Seven thousand three hundred ninety-five.
17. Fifty-seven billions fifty-nine millions ninety-nine thousand and forty-seven.

QUESTIONS. — Art. 13. What is the rule for *writing* numbers according to the French method? At which hand do you begin to numerate figures? Where do you begin to read them? At which hand do you begin to write numbers? Why?

Art. 14. The following table exhibits the English method of numeration, in which it will be observed that the figures are separated by commas into periods of six figures each. The first or right-hand period is regarded as units and thousands of units; the second, as millions and thousands of millions; and so on, six places being assigned to each period designated by a distinct name.

ENGLISH NUMERATION TABLE.

Hund. of Thousands of Trillions.	Tens of Thousands of Trillions.	Thousands of Trillions.	Hundreds of Trillions.	Tens of Trillions.	Trillions.	Hund. of Thousands of Billions.	Tens of Thousands of Billions.	Thousands of Billions.	Hundreds of Billions.	Tens of Billions.	Billions.	Hund. of Thousands of Millions.	Tens of Thousands of Millions.	Thousands of Millions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
1	3	7	8	9	0,	7	1	1	7	1	6,	3	7	1	7	1	2,	4	5	6	7	1	1.
Period of Trillions.						Period of Billions.						Period of Millions.						Period of Units.					

The value of the figures in the above table, expressed in words according to the English method, is, One hundred thirty-seven thousand eight hundred ninety trillions; seven hundred eleven thousand seven hundred sixteen billions; three hundred seventy-one thousand seven hundred twelve millions; four hundred fifty-six thousand seven hundred eleven.

Although there is the same number of figures in the English and in the French table, yet it will be observed that in the French table we have the names of three periods other than in the English. It will also be observed that the variation commences after the ninth place, or the place of hundreds of millions. If, therefore, we would know the value of numbers

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Questions. — Art. 14. How many figures in each period in the English method of numeration? What orders are found in the English method that are not in the French? Give the names of the periods in the English Numeration Table, beginning with the period of units. Repeat the table, giving the names of all the orders or places. What is the value of the numbers in the table expressed in words? How do the figures in the English and French table compare as to numbers? How as to periods? Why is this difference? Has a million the same value reckoned by the French table as when reckoned by the English?

higher than hundreds of millions, when we see them written in words, or hear them read, we need to know whether they are expressed according to the French or the English method of numeration.

The English method of numeration is that generally used in Great Britain, and in the British Provinces.

ART. **15.** To numerate and read numbers expressed by figures according to the English method, we have the following

RULE. — *Begin at the right hand, and point off the figures into periods of* SIX *places each. Then, commencing at the left hand, read the figures of each period, adding the name of each period, excepting that of units.*

EXERCISES IN ENGLISH NUMERATION.

The learner may read orally, or write in words, the following numbers:

1.	125	5.	27306387903
2.	1063	6.	531470983712
3.	25842	7.	4230578032765038
4.	904357	8.	71675637880737076708638970647 3

ART. **16.** To write numbers in figures, according to the English method, we have the following

RULE. — *Begin at the left hand, and write in each successive order the figure belonging to it.*

If any intervening order would otherwise be vacant, fill the place by a cipher.

EXERCISES IN ENGLISH NOTATION AND NUMERATION.

The learner may write in figures, and read, the following numbers:

1. Three hundred twenty-five thousand four hundred and twelve.

2. Two hundred fourteen thousand, one hundred sixty-five millions, seventy-eight thousand and fifty-six.

3. Forty-two billions, six hundred seventeen thousand thirty-one millions, forty-one thousand three hundred forty-two.

4. Two thousand eight billions, nine thousand eighty-two millions, seven hundred one thousand, nine hundred and eight.

QUESTIONS. — Has the billion the same value as that by the French table? Why not? By which table has it the greater value? Where is the English method of numerating in use? — Art. 15. What is the rule for numerating and reading numbers by the English method? — Art. 16. What is the rule for *writing* numbers according to the English method?

§ II. ADDITION.

MENTAL EXERCISES.

ART. 17. WHEN it is required to find a single number to express the sum of the units contained in several smaller numbers, the process is called *Addition.*

Ex. 1. James has 3 pears, and his younger brother has 4; how many have both?

ILLUSTRATION. — To solve this question, the 3 pears and 4 pears must be added together; thus, 3 added to 4 makes 7. Therefore James and his brother have 7 pears.

ADDITION TABLE.

2 and 0 are 2	3 and 0 are 3	4 and 0 are 4	5 and 0 are 5
2 and 1 are 3	3 and 1 are 4	4 and 1 are 5	5 and 1 are 6
2 and 2 are 4	3 and 2 are 5	4 and 2 are 6	5 and 2 are 7
2 and 3 are 5	3 and 3 are 6	4 and 3 are 7	5 and 3 are 8
2 and 4 are 6	3 and 4 are 7	4 and 4 are 8	5 and 4 are 9
2 and 5 are 7	3 and 5 are 8	4 and 5 are 9	5 and 5 are 10
2 and 6 are 8	3 and 6 are 9	4 and 6 are 10	5 and 6 are 11
2 and 7 are 9	3 and 7 are 10	4 and 7 are 11	5 and 7 are 12
2 and 8 are 10	3 and 8 are 11	4 and 8 are 12	5 and 8 are 13
2 and 9 are 11	3 and 9 are 12	4 and 9 are 13	5 and 9 are 14
2 and 10 are 12	3 and 10 are 13	4 and 10 are 14	5 and 10 are 15
2 and 11 are 13	3 and 11 are 14	4 and 11 are 15	5 and 11 are 16
2 and 12 are 14	3 and 12 are 15	4 and 12 are 16	5 and 12 are 17
6 and 0 are 6	7 and 0 are 7	8 and 0 are 8	9 and 0 are 9
6 and 1 are 7	7 and 1 are 8	8 and 1 are 9	9 and 1 are 10
6 and 2 are 8	7 and 2 are 9	8 and 2 are 10	9 and 2 are 11
6 and 3 are 9	7 and 3 are 10	8 and 3 are 11	9 and 3 are 12
6 and 4 are 10	7 and 4 are 11	8 and 4 are 12	9 and 4 are 13
6 and 5 are 11	7 and 5 are 12	8 and 5 are 13	9 and 5 are 14
6 and 6 are 12	7 and 6 are 13	8 and 6 are 14	9 and 6 are 15
6 and 7 are 13	7 and 7 are 14	8 and 7 are 15	9 and 7 are 16
6 and 8 are 14	7 and 8 are 15	8 and 8 are 16	9 and 8 are 17
6 and 9 are 15	7 and 9 are 16	8 and 9 are 17	9 and 9 are 18
6 and 10 are 16	7 and 10 are 17	8 and 10 are 18	9 and 10 are 19
6 and 11 are 17	7 and 11 are 18	8 and 11 are 19	9 and 11 are 20
6 and 12 are 18	7 and 12 are 19	8 and 12 are 20	9 and 12 are 21
10 and 0 are 10	11 and 0 are 11	12 and 0 are 12	13 and 0 are 13
10 and 1 are 11	11 and 1 are 12	12 and 1 are 13	13 and 1 are 14
10 and 2 are 12	11 and 2 are 13	12 and 2 are 14	13 and 2 are 15
10 and 3 are 13	11 and 3 are 14	12 and 3 are 15	13 and 3 are 16
10 and 4 are 14	11 and 4 are 15	12 and 4 are 16	13 and 4 are 17
10 and 5 are 15	11 and 5 are 16	12 and 5 are 17	13 and 5 are 18
10 and 6 are 16	11 and 6 are 17	12 and 6 are 18	13 and 6 are 19
10 and 7 are 17	11 and 7 are 18	12 and 7 are 19	13 and 7 are 20
10 and 8 are 18	11 and 8 are 19	12 and 8 are 20	13 and 8 are 21
10 and 9 are 19	11 and 9 are 20	12 and 9 are 21	13 and 9 are 22
10 and 10 are 20	11 and 10 are 21	12 and 10 are 22	13 and 10 are 23
10 and 11 are 21	11 and 11 are 22	12 and 11 are 23	13 and 11 are 24
10 and 12 are 22	11 and 12 are 23	12 and 12 are 24	13 and 12 are 25

QUESTION. — Art. 17. What is the process called by which we find the sum of several numbers?

2. How many are 2 and 3? 2 and 5? 2 and 7? 2 and 9? 2 and 4? 2 and 2? 2 and 8? 2 and 6?

3. How many are 3 and 3? 3 and 5? 3 and 7? 3 and 9? 3 and 4? 3 and 6? 3 and 8? 3 and 3?

4. How many are 4 and 3? 4 and 5? 4 and 8? 4 and 9? 4 and 1? 4 and 2? 4 and 4? 4 and 7?

5. How many are 5 and 3? 5 and 4? 5 and 7? 5 and 8? 5 and 9? 5 and 2? 5 and 5? 5 and 6? 5 and 1?

6. How many are 6 and 2? 6 and 4? 6 and 3? 6 and 5? 6 and 7? 6 and 9? 6 and 1? 6 and 6? 6 and 8?

7. How many are 7 and 3? 7 and 5? 7 and 7? 7 and 6? 7 and 8? 7 and 9? 7 and 2? 7 and 4? 7 and 10?

8. How many are 8 and 2? 8 and 4? 8 and 5? 8 and 7? 8 and 9? 8 and 8? 8 and 1? 8 and 3? 8 and 6?

9. How many are 9 and 1? 9 and 3? 9 and 5? 9 and 4? 9 and 6? 9 and 8? 9 and 9? 9 and 2?

10. James had 4 apples, Samuel gave him 5 more, and John gave him 6; how many had he in all?

11. Gave 7 dollars for a barrel of flour, 5 dollars for a hundred weight of sugar, and 8 dollars for a tub of butter; what did I give for the whole?

12. Paid 5 dollars for a pair of boots, 12 dollars for a coat, and 6 dollars for a vest; what was the whole cost?

13. Gave 25 cents for an arithmetic, and 67 for a geography; what was the cost of both?

ILLUSTRATION. — We may divide the cents into tens and units. Thus, 25 equals 2 tens and 5 units; 67 equals 6 tens and 7 units; 2 tens and 6 tens are 8 tens; and 5 units and 7 units are 12 units, or 1 ten and 2 units; 1 ten and 2 units added to 8 tens make 9 tens and 2 units, or 92. Therefore the arithmetic and geography cost 92 cents.

14. On the fourth of July 20 cents were given to Emily, 15 cents to Betsey, 10 cents to Benjamin, and none to Lydia; what did they all receive?

15. Bought four loads of hay; the first cost 15 dollars, the second 12 dollars, the third 20 dollars, and the fourth 17 dollars; what was the price of the whole?

16. Gave 55 dollars for a horse, 40 dollars for a wagon, and 17 dollars for a harness; what did they all cost?

17. Sold 3 loads of wood for 17 dollars, 6 tons of timber for 19 dollars, and a pair of oxen for 60 dollars; what sum did I receive?

ART. 18. From the solution of the preceding questions, the learner will perceive, that

ADDITION is the process of finding the sum of two or more numbers. The result obtained, is called their *amount.*

Addition is commonly represented by this character, +, which signifies *plus*, or added to. The expression 7+5 is read, 7 plus 5, or 7 added to 5.

This character, =, is called the sign of equality, and signifies *equal to.* The expression 7+5=12 is read, 7 plus 5, or 7 added to 5, is equal to 12.

EXERCISES FOR THE SLATE.

ART. 19. The method of operation when the numbers are large, and the sum of each column is less than 10.

EX. 1. A man bought a watch for 42 dollars, a coat for 16 dollars, and a set of maps for 21 dollars; what did he pay for the whole? Ans. 79 dollars.

OPERATION.
Dollars.
42
16
21
Amount 79

Having arranged the numbers so that all the units of the same order shall stand in the same column, we first add the column of *units;* thus, 1 and 6 are 7, and 2 are 9 (units), and write down the amount under the column of units. We then add the column of *tens;* thus, 2 and 1 are 3, and 4 are 7 (tens), which we write under the column of tens, and thus find the amount of the whole to be 79 dollars.

ART. 20. *First Method of Proof.* — Begin at the top and add the columns downward in the same manner as they were before added upward, and if the two sums agree the work is presumed to be right.

The reason of this proof is, that, by adding downward, the order of the figures is inverted; and, therefore, any error made in the first addition would probably be detected in the second.

NOTE. — This method of proof is generally used in business.

QUESTIONS. — Art. 18. What is addition? What is the sign of addition, and what does it signify? What is the sign of equality, and what does it signify? — Art. 19. How are numbers arranged for addition? Which column must first be added? Why? Where do you place its sum? Where must the sum of each column be placed? What is the whole sum called? — Art. 20. How is addition proved? What is the reason for this method of proof? Is this method in common use?

EXAMPLES FOR PRACTICE.

2.	3.	4.	5.
Miles.	Furlongs.	Days.	Weeks.
151	234	472	121
212	423	315	516
321	321	102	361
Ans. 684			

6. What is the sum of 231, 114, and 324? Ans. 669.

7. Required the sum of 235, 321, and 142.

8. What is the sum of 11, 22, 505, and 461?

9. Sold twelve ploughs for 104 dollars, two wagons for 214 dollars, and one chaise for 121 dollars; what was the amount of the whole?

10. A drover bought 125 sheep of one man, 432 of another, and of a third 311; how many did he buy?

ART. 21. Method of operation when the sum of any column is equal to or exceeds 10.

Ex. 1. I have three lots of wild land; the first contains 246 acres, the second 764 acres, and the third 918 acres. I wish to know how many acres are in the three lots. Ans. 1928 acres.

OPERATION.
Acres.
246
764
918
Amount 1928

Having arranged the numbers as in the preceding examples, we first add the units; thus, 8 and 4 are 12, and 6 are 18 units, equal 1 ten and 8 units. We write the 8 units under the column of units, we *carry* or add the 1 ten to the column of tens; thus, 1 added to 1 makes 2, and 6 are 8, and 4 are 12 (tens), equal to 1 hundred and 2 tens. We write the 2 tens under the column of tens, and add the 1 hundred to the column of hundreds; thus, 1 added to 9 makes 10, and 7 are 17, and 2 are 19 (hundreds), equal to 1 thousand and 9 hundreds. We write the 9 under the column of hundreds; and there being no other column to be added, we set down the 1 thousand in thousands' place, and find the amount of the several numbers to be 1928.

NOTE. — A more concise way, in practice, is to omit calling the name of each figure as added, and name only results.

QUESTIONS. — Art. 21. When the sum of any column exceeds ten, where are the units written? What is done with the tens? Why do you carry, or add, one for every 10? How is the sum of the last column written?

ART. **22.** From the preceding examples and illustrations in addition, we deduce the following general

RULE. — *Write the numbers so that all the figures of the same order shall stand in the same column.*

Add, upward, all the figures in the column of units, and, if the amount be less than ten, write it underneath. But, if the amount be ten or more, write down the unit figure only, and add in the figure denoting the ten or tens with the next column.

Proceed in this way with each column, until all are added, observing to write down the whole amount of the last column.

ART. **23.** *Second Method of Proof.* — Separate the numbers to be added into two parts, by drawing a horizontal line between them. Add the numbers below the line, and set down their sum. Then add this sum and the number, or numbers, above the line together; and, if their sum is equal to the first amount, the work is presumed to be right.

The reason of this proof depends on the principle, *That the sum of all the parts into which any number is divided is equal to the whole.*

EXAMPLES FOR PRACTICE.

2. OPERATION.	2. OPERATION AND PROOF.	3. OPERATION.	3. OPERATION AND PROOF.
526	526	241	241
317		532	
529	317	207	532
132	529	913	207
	132		913
Ans. 1504		Ans. 1893	
	First am't 1504		First am't 1893
	978		1652
	Ans. 1504		Ans. 1893

4. Dollars.	5. Miles.	6. Pounds.	7. Rods.	8. Inches.	9. Feet.
11	47	127	678	789	1769
23	87	396	971	478	7895
97	58	787	147	719	7563
86	83	456	716	937	8765
217	275	1766	2512	2923	25992

QUESTIONS. — Art 22. What is the general rule for addition? — Art. 23. *What is the second* method of proving addition? What is the reason of this *method of proof?*

10. Ounces.	11. Drams.	12. Cents.	13. Eagles.	14. Degrees.
876	789	123	471	1234
376	567	478	617	3456
715	743	716	871	6544
678	435	478	317	7891
910	678	127	899	8766

15. Feet.	16. Inches.	17. Hours.	18. Minutes.
78956	71678	71123	98765
37667	12345	45678	12345
12345	67890	34680	67111
67890	34567	56777	33333
78999	89012	67812	71345
13579	78917	71444	99999

19. Days.	20. Years.	21. Months.	22. Hogsheads.
17875897	789567	37	30176
71675 12	7613	1378956	31
876567	123123	700714	8601
98765	70071	367	11
7896	475	76117	9911
789	1069	4611779	89120
78	374176	9171	710
7	761	131765	4325

23. Add 1001, 76, 10078, 15, 8761, 7, and 1678.

24. Add 49, 761, 3756, 8, 150, 761761, and 18.

25. Required the sum of 3717, 8, 7, 10001, 58, 18, and 5.

26. Add 19, 181, 5, 897156, 81, 800, and 71512.

27. What is the sum of 999, 8081, 9, 1567, 88, 91, 7, and 878?

28. Add 71, 18765, 9111, 1471, 678, 9, 1446, and 71.

29. Add 51, 1, 7671, 89, 871787, 61, and 70001.

30. What is the sum of 71, 8956, 1, 785, 587, and 76178?

31. Add 9999, 8008, 8, 81, 4777, and 516785.

32. Add 5, 7, 8911, 467, 47895, and 87.
33. Add 123456, 71, 8005, 21, and 716787.

34. Add 47, 911111, 717, 81, 88767, and 56.

35. What is the sum of 71, 8899, 4, 7111, and 678679?

36. Add 81, 879, 41, 76789, 42, 1, and 78967.

37. Add 917658, 75, 876789, 46, and 8222.

38. Add 91, 76756895, 76, 14, 3, and 76378.

39. Add 10, 100, 1000, 10000, 100000, and 1000000.

40. What is the sum of 9, 99, 99, 1111, 8000, and 5?

41. Add 41, 7651, 7678956, 43, 15, and 6780.

42. Add 1234, 7891, 3146751, 27, 9, and 5.

43. What is the sum of 19, 91, 1, 1, 1478, 1007, and 46?

44. Add four hundred seventy-six, seventy-one, one hundred five, and three hundred eighty-seven.

45. Add fifty-six thousand seven hundred eighty-five, seven hundred five, thirty-six, one hundred seventy thousand and one, and four hundred seven.

46. Add fifty-six thousand seven hundred eleven, three thousand seventy-one, four hundred seventy-one, sixty-one, and three thousand and one.

47. What is the sum of the following numbers: seven hundred thousand seven hundred one, seventeen thousand nine, one million six hundred thousand seven hundred six, forty-seven thousand six hundred seventy-one, seven thousand forty-seven, four hundred one, and nine?

48. Gave 73 dollars for a watch, 15 dollars for a cane, 119 dollars for a horse, 376 dollars for a carriage, and 7689 dollars *for a house; how* much did they all cost?

49. In an orchard, 15 trees bear plums, 73 trees bear apples, 29 trees bear pears, and 14 trees bear cherries; how many trees are there in the orchard?

50. The hind quarters of an ox weighed 375 pounds each, the fore quarters 315 pounds each; the hide weighed 96 pounds, and the tallow 87 pounds. What was the whole weight of the ox?

51. A man bought a farm for 1728 dollars, and sold it so as to gain 375 dollars; how much did he sell it for?

52. A merchant bought five pieces of cloth. For the first he gave 376 dollars, for the second 198 dollars, for the third 896 dollars, for the fourth 691 dollars, and for the fifth 96 dollars. How much did he give for the whole?

53. A merchant bought five hogsheads of molasses for 375 dollars, and sold it so as to gain 25 dollars on each hogshead; for how much did he sell it?

54. John Smith's farm is worth 7896 dollars; he has bank stock valued at 369 dollars, and he has in cash 850 dollars. How much is he worth?

55. Required the number of inhabitants in the New England States. By the census of 1850 there were in Maine 583,169, in New Hampshire 317,976, in Massachusetts 994,514, in Rhode Island 147,545, in Connecticut 370,792, and in Vermont 314,120.

56. Required the number of inhabitants in the Middle States, including the District of Columbia. In 1850 there were in New York 3,097,394, in New Jersey 489,555, in Pennsylvania 2,311,786, in Delaware 91,532, in Maryland 583,034, and in the District of Columbia 51,687.

57. Required the number of inhabitants in the Southern States. In 1850 there were in Virginia 1,421,661, in North Carolina 869,039, in South Carolina 668,507, in Georgia 906,185, and in Florida 87,445.

58. Required the number of inhabitants in the South-Western States. In 1850 there were in Alabama 771,623, in Mississippi 606,526, in Louisiana 517,762, in Texas 212,592, in Arkansas 209,897, and in Tennessee 1,002,917.

59. Required the number of inhabitants in the North-Western States and Territories. In 1850 there were in Missouri 682,044, in Kentucky 982,405, in Ohio 1,980,329, in Indiana 988,416, in Illinois 851,470, in Michigan 397,654, in Wisconsin 305,391,

in Iowa 192,214, in California 92,597, and in the Territories 92,298.

ART. 24. Method of adding two or more columns at a single operation.

Ex. 1. Washington lived 68 years; John Adams, 91 years Jefferson, 83 years; Madison, 85 years. What is the sum of the years they all lived? Ans. 327.

OPERATION.

Years.
68
91
83
85
Amount 327

Beginning with the number last written down, we add the units and tens, thus: 85 and 3 equal 88, and 80 equal 168, and 1 equal 169, and 90 equal 259, and 8 equal 267, and 60 equal 327, the sum sought. In like manner may be added more than two columns at one operation.

NOTE. — The examples that follow can be performed as the above, or by the common method, or by both, as the teacher may advise.

2.	3.	4.	5.	6.
Ounces.	Yards.	Feet.	Inches.	Chaldrons.
1234	2345	3456	7891	5678
5678	6789	7891	1356	3215
9012	1023	3456	7891	6789
3456	4456	7891	2345	3214
7890	7890	3456	6789	1234
1345	1234	7890	1234	3789
6789	5678	1378	5678	1379
3216	9012	8123	9123	9008
7890	3456	4567	4567	1071
1030	7890	8912	8912	7163
7055	1345	3456	3456	6781
5678	6789	7891	7812	1780
1234	3456	3456	3456	3007
5678	7890	7891	7812	5617
9001	5678	3783	3713	4456
2345	9012	1237	7891	3456
6789	3456	7891	1357	7891
1030	7890	1007	9009	3070
7816	1234	5670	8765	4567
1781	5678	1234	4321	3456

§ III. SUBTRACTION.

MENTAL EXERCISES.

ART. **25.** WHEN it is required to find the difference between two numbers, the process is called *Subtraction.* The operation is the reverse of addition.

Ex. 1. John has 7 oranges, and his sister but 4; how many more has John than his sister?

ILLUSTRATION. — We first inquire what number added to 4 will make 7. From addition we learn that 4 and 3 are 7; consequently, if 4 oranges be taken from 7 oranges, 3 will remain, Hence John has 3 oranges more than his sister.

SUBTRACTION TABLE.

1 from 1 leaves 0	2 from 2 leaves 0	3 from 3 leaves 0	4 from 4 leaves 0
1 from 2 leaves 1	2 from 3 leaves 1	3 from 4 leaves 1	4 from 5 leaves 1
1 from 3 leaves 2	2 from 4 leaves 2	3 from 5 leaves 2	4 from 6 leaves 2
1 from 4 leaves 3	2 from 5 leaves 3	3 from 6 leaves 3	4 from 7 leaves 3
1 from 5 leaves 4	2 from 6 leaves 4	3 from 7 leaves 4	4 from 8 leaves 4
1 from 6 leaves 5	2 from 7 leaves 5	3 from 8 leaves 5	4 from 9 leaves 5
1 from 7 leaves 6	2 from 8 leaves 6	3 from 9 leaves 6	4 from 10 leaves 6
1 from 8 leaves 7	2 from 9 leaves 7	3 from 10 leaves 7	4 from 11 leaves 7
1 from 9 leaves 8	2 from 10 leaves 8	3 from 11 leaves 8	4 from 12 leaves 8
1 from 10 leaves 9	2 from 11 leaves 9	3 from 12 leaves 9	4 from 13 leaves 9
1 from 11 leaves 10	2 from 12 leaves 10	3 from 13 leaves 10	4 from 14 leaves 10
1 from 12 leaves 11	2 from 13 leaves 11	3 from 14 leaves 11	4 from 15 leaves 11
1 from 13 leaves 12	2 from 14 leaves 12	3 from 15 leaves 12	4 from 16 leaves 12
5 from 5 leaves 0	6 from 6 leaves 0	7 from 7 leaves 0	8 from 8 leaves 0
5 from 6 leaves 1	6 from 7 leaves 1	7 from 8 leaves 1	8 from 9 leaves 1
5 from 7 leaves 2	6 from 8 leaves 2	7 from 9 leaves 2	8 from 10 leaves 2
5 from 8 leaves 3	6 from 9 leaves 3	7 from 10 leaves 3	8 from 11 leaves 3
5 from 9 leaves 4	6 from 10 leaves 4	7 from 11 leaves 4	8 from 12 leaves 4
5 from 10 leaves 5	6 from 11 leaves 5	7 from 12 leaves 5	8 from 13 leaves 5
5 from 11 leaves 6	6 from 12 leaves 6	7 from 13 leaves 6	8 from 14 leaves 6
5 from 12 leaves 7	6 from 13 leaves 7	7 from 14 leaves 7	8 from 15 leaves 7
5 from 13 leaves 8	6 from 14 leaves 8	7 from 15 leaves 8	8 from 16 leaves 8
5 from 14 leaves 9	6 from 15 leaves 9	7 from 16 leaves 9	8 from 17 leaves 9
5 from 15 leaves 10	6 from 16 leaves 10	7 from 17 leaves 10	8 from 18 leaves 10
5 from 16 leaves 11	6 from 17 leaves 11	7 from 18 leaves 11	8 from 19 leaves 11
5 from 17 leaves 12	6 from 18 leaves 12	7 from 19 leaves 12	8 from 20 leaves 12
9 from 9 leaves 0	10 from 10 leaves 0	11 from 11 leaves 0	12 from 12 leaves 0
9 from 10 leaves 1	10 from 11 leaves 1	11 from 12 leaves 1	12 from 13 leaves 1
9 from 11 leaves 2	10 from 12 leaves 2	11 from 13 leaves 2	12 from 14 leaves 2
9 from 12 leaves 3	10 from 13 leaves 3	11 from 14 leaves 3	12 from 15 leaves 3
9 from 13 leaves 4	10 from 14 leaves 4	11 from 15 leaves 4	12 from 16 leaves 4
9 from 14 leaves 5	10 from 15 leaves 5	11 from 16 leaves 5	12 from 17 leaves 5
9 from 15 leaves 6	10 from 16 leaves 6	11 from 17 leaves 6	12 from 18 leaves 6
9 from 16 leaves 7	10 from 17 leaves 7	11 from 18 leaves 7	12 from 19 leaves 7
9 from 17 leaves 8	10 from 18 leaves 8	11 from 19 leaves 8	12 from 20 leaves 8
9 from 18 leaves 9	10 from 19 leaves 9	11 from 20 leaves 9	12 from 21 leaves 9
9 from 19 leaves 10	10 from 20 leaves 10	11 from 21 leaves 10	12 from 22 leaves 10
9 from 20 leaves 11	10 from 21 leaves 11	11 from 22 leaves 11	12 from 23 leaves 11
9 from 21 leaves 12	10 from 22 leaves 12	11 from 23 leaves 12	12 from 24 leaves 12

QUESTIONS. — Art. 25. What does subtraction teach? Of what is it the reverse?

2. Thomas had five oranges, and gave two of them to John · how many had he left?

3. Peter had six marbles, and gave two of them to Samuel, how many had he left?

4. Lydia had four cakes; having lost one, how many had she left?

5. Daniel, having eight cents, gives three to Mary; how many has he left?

6. Benjamin had ten nuts; he gave four to Jane, and three to Emily; how many had he left?

7. Moses gives eleven oranges to John, and eight to Enoch; how many more has John than Enoch?

8. Paid seven dollars for a pair of boots, and two dollars for shoes; how much did the boots cost more than the shoes?

9. How many are 4 less 2? 4 less 1? 4 less 4?

10. How many are 4 less 3? 5 less 1? 5 less 5?

11. How many are 5 less 2? 5 less 3? 5 less 4?

12. How many are 6 less 1? 6 less 2? 6 less 4? 6 less 5?

13. How many are 7 less 2? 7 less 3? 7 less 4? 7 less 6?

14. How many are 8 less 6? 8 less 5? 8 less 2? 8 less 4? 8 less 1?

15. How many are 9 less 2? 9 less 4? 9 less 5? 9 less 7? 9 less 3?

16. How many are 10 less 8? 10 less 7? 10 less 5? 10 less 3? 10 less 1?

17. How many are 11 less 9? 11 less 7? 11 less 5? 11 less 3? 11 less 4?

18. How many are 12 less 10? 12 less 8? 12 less 6? 12 less 4? 12 less 7?

19. How many are 13 less 11? 13 less 10? 13 less 7? 13 less 9? 13 less 5?

20. How many are 14 less 11? 14 less 9? 14 less 8? 14 less 6? 14 less 7? 14 less 3?

21. How many are 15 less 2? 15 less 4? 15 less 5? 15 less 7? 15 less 9? 15 less 13?

22. How many are 16 less 3? 16 less 4? 16 less 7? 16 less 9? 16 less 11? 16 less 15?

23. How many are 17 less 1? 17 less 3? 17 less 5? 17 less 7? 17 less 8? 17 less 12?

24. How many are 18 less 2? 18 less 4? 18 less 7? 18 less 8? 18 less 10? 18 less 12?

25. How many are 19 less 1? 19 less 3? 19 less 5? 19 less 7? 19 less 9? 19 less 16?

26. How many are 20 less 5? 20 less 8? 20 less 9? 20 less 12? 20 less 15? 20 less 19?

27. Bought a horse for 60 dollars, and sold him for 90 dollars; how much did I gain?

ILLUSTRATION. — We may divide the two prices of the horse into tens, and subtract the greater from the less. Thus 60 equals 6 tens, and 90 equals 9 tens; 6 tens from 9 tens leave 3 tens, or 30. Therefore I gained 30 dollars.

28. Sold a wagon for 70 dollars, which cost me 100 dollars; how much did I lose?

29. John travels 30 miles a day, and Samuel 90 miles; what is the difference?

30. I have 100 dollars, and after I shall have given 20 to Benjamin, and paid a debt of 30 dollars to J. Smith, how many dollars have I left?

31. John Smith, Jr., had 170 dollars; he gave his oldest daughter, Angeline, 40 dollars, his youngest daughter, Mary, 50 dollars, his oldest son, James, 30, and his youngest son, William, 20 dollars; he also paid 20 dollars for his taxes; how many dollars had he remaining?

ART. **26.** The pupil, having solved the preceding questions, will perceive, that

SUBTRACTION is the taking of one number from another to find the difference.

When the two numbers are unequal, the larger is called the *Minuend*, and the less number the *Subtrahend*. The answer, or number found by the operation, is called the *Difference*, or *Remainder*.

NOTE. — The words *minuend* and *subtrahend* are derived from two Latin words; the former from *minuendum*, which signifies *to be diminished* or *made less*, and the latter from *subtrahendum*, which means *to be subtracted* or *taken away*.

ART. **27.** SIGNS. — Subtraction is denoted by a short horizontal line, thus —, signifying *minus*, or *less*. It indicates that the number *following* is to be taken from the one that *precedes* it. The expression $6 - 2 = 4$ is read, 6 minus, or less, 2 is equal to 4.

QUESTIONS: — Art. 26. What is subtraction? What is the greater number called? What is the less number called? What the answer? — Art. 27. What is the sign of subtraction? What does it signify and indicate?

Exercises for the Slate.

Art. 28. Method of operation, when the numbers are large and each figure in the subtrahend is less than the figure above it in the minuend.

Ex. 1. Let it be required to take 245 from 468, and to find their difference. Ans. 223.

	Operation.
Minuend	4 6 8
Subtrahend	2 4 5
Remainder	2 2 3

We place the less number under the greater, units under units, tens under tens, &c., and draw a line below them. We then begin at the right hand, and say, 5 units from 8 units leave 3 units, and write the 3 in units' place below. We then say, 4 tens from 6 tens leave 2 tens, and write the 2 in tens' place below; and proceed with the next figure, and say, 2 hundreds from 4 hundreds leave 2 hundreds, which we write in hundreds' place below. We thus find the difference to be 223.

Art. 29. *First Method of Proof.* — Add the remainder and the subtrahend together, and their sum will be equal to the minuend, if the work is right.

This method of proof depends on the principle, *That the greater of any two numbers is equal to the less added to the difference between them.*

Examples for Practice.

	2. Operation.	2. Operation and Proof.	3. Operation.	3. Operation and Proof.
Minuend	5 4 7	5 4 7	9 8 6	9 8 6
Subtrahend	2 3 5	2 3 5	7 6 3	7 6 3
Remainder	3 1 2	3 1 2	2 2 3	2 2 3
		Min. 5 4 7		Min. 9 8 6

	4.	5.	6.	7.
From	6 8 4	7 3 5	8 6 4	9 4 8
Take	4 6 2	5 2 3	6 5 1	7 4 6

8. A farmer paid 539 dollars for a span of fine horses, and sold them for 425 dollars; how much did he lose?

9. A farmer raised 896 bushels of wheat, and sold 675 bushels of it; how much did he reserve for his own use?

Questions. — Art. 28. How are numbers arranged for subtraction? Where do you begin to subtract? Why? Where do you write the difference? — Art. 29. What is the first method of proving subtraction? What is the reason of this proof, or on what principle does it depend?

10. A gentleman gave his son 3692 dollars, and his daughter 1212 dollars less than his son; how much did his daughter receive?

ART. **30.** Method of operation when any figure in the subtrahend is greater than the figure above it in the minuend.

Ex. 1. If I have 624 dollars, and lose 342 of them, how many remain? Ans. 282.

OPERATION.

Minuend	6 2 4
Subtrahend	3 4 2
Remainder	2 8 2

We first take the 2 units from the 4 units, and find the difference to be 2 units, which we write under the figure subtracted. We then proceed to take the 4 tens from the 2 tens above it; but we here find a difficulty, since the 4 is greater than 2, and cannot be subtracted from it. We therefore add 10 to the 2 tens, which makes 12 tens, and then subtract the 4 from 12, and 8 tens remain, which we write below. Then, to compensate for the 10 thus added to the 2 in the minuend, we add one to the 3 hundreds in the next higher place in the subtrahend, which makes 4 hundreds, and subtract the 4 from 6 hundreds, and 2 hundreds remain. The remainder, therefore, is 282.

The reason of this operation depends upon the self-evident truth, *That, if any two numbers are equally increased, their difference remains the same.* In this example 10 tens, equal to 1 hundred, were added to the 2 tens in the upper number, and 1 was added to the 2 hundreds in the lower number. Now, since the 3 stands in the hundreds' place, the 1 added was in fact 1 hundred. Hence, the two numbers being equally increased, the difference is the same.

NOTE. — This addition of 10 to the minuend is sometimes called *borrowing* 10, and the addition of 1 to the subtrahend is called *carrying* 1.

ART. **31.** From the preceding examples and illustrations in subtraction, we deduce the following general

RULE. — *Place the less number under the greater, so that units of the same order shall stand in the same column.*

Commencing at the right hand, subtract each figure of the subtrahend from the figure above it.

If any figure of the subtrahend is larger than the figure above it in the minuend, add 10 *to that figure of the minuend before subtracting, and then add* 1 *to the next figure of the subtrahend.*

QUESTIONS. — Art. 30. How do you proceed when a figure of the subtrahend is larger than the one above it in the minuend? How do you compensate for the 10 which is added to the minuend? What is the reason for this addition to the minuend and subtrahend? How does it appear that the 1 added to the subtrahend equals the 10 added to the minuend? What is the addition of 10 to the minuend sometimes called? The addition of 1 to the subtrahend? — Art. 31. What is the general rule for subtraction?

Art. 32. *Second Method of Proof.* — Subtract the remainder or difference from the minuend, and the result will be like the subtrahend if the work is right.

This method of proof depends on the principle, *That the smaller of any two numbers is equal to the remainder obtained by subtracting their difference from the greater.*

Examples for Practice.

	2. Operation.	2. Operation and Proof.	3. Operation.	3. Operation and Proof.
Minuend	376	376	531	531
Subtrahend	167	167	389	389
Remainder	209	209	142	142
		Sub. 167		Sub. 389

	4. Tons.	5. Gallons.	6. Pecks.	7. Feet.
From	978	67158	14711	100000
Take	199	14339	9197	90909
Ans.	779	52819	5514	9091

	8. Miles.	9. Dollars.	10. Minutes.	11. Seconds.
From	67895	456798	765321	555555
Take	19999	190899	177777	177777

	12. Rods.	13. Acres.
From	1002003004400500	1000000000000
Take	908070605040 30	999999999999

14. From 671111 take 199999.
15. From 1789100 take 808088.
16. From 1000000 take 999999.
17. From 9999999 take 1607.
18. From 6101507601061 take 3806790989.

Questions. — Art. 32. What is the second method of proving subtraction? What is the reason for this method of proof, or on what principle does it *depend?*

19. From 8054010657811 take 76909748598.

20. From 7100071641115 take 10071178.

21. From 501505010678 take 794090589.

22. Take 99999999 from 100000000.
23. Take 44444444 from 500000000.
24. Take 1234567890 from 9987654321.

25. From 800700567 take 1010101.
26. Take twenty-five thousand twenty-five from twenty-five millions.
27. Take nine thousand ninety-nine from ninety-nine thousand.
28. From one hundred one millions ten thousand one hundred one take ten millions one hundred one thousand and ten.

29. From one million take nine.
30. From three thousand take thirty-three.
31. From one hundred millions take five thousand.

32. From 1,728 dollars, I paid 961 dollars; how many remain?
33. Our national independence was declared in 1776; how many years from that period to the close of the last war with Great Britain, in 1815?
34. The last transit of Venus was in 1769, and the next will be in 1874; how many years will intervene?

Ans. 105 years.

35. The State of New Jersey contains 6851 square miles, and Delaware 2120. How many more square miles has the former State than the latter?
36. In 1840 the number of inhabitants in the United States was 17,069,453, and in 1850 it was 23,191,876; what was the increase?
37. In 1850 there were raised in the State of Ohio 56,619,608 bushels of corn, and in 1853, 73,436,690 bushels; what was the increase?
38. By the census of 1850, 13,121,498 bushels of wheat were raised in New York, and 15,367,691 bushels in Pennsylvania; how many bushels in the latter State more than in the former?

39. The city of New York owes 13,960,856 dollars, and Boston owes 7,779,855 dollars; how much more does New York owe than Boston?

40. From five hundred eighty-one thousand take three thousand and ninety-six.

41. It was ascertained by a transit of Venus, June 3, 1769, that the mean distance of the earth from the sun was ninety-five millions one hundred seventy-three thousand one hundred twenty-seven miles, and that the mean distance of Mars from the sun was one hundred forty-five millions fourteen thousand one hundred forty-eight miles. Required the difference of their distances from the sun.

ART. **33.** Method of subtracting when there are two or more subtrahends.

EX. 1. A man owing 767 dollars, paid at one time 190 dollars, at another time 131 dollars, at another time 155 dollars; how much did he then owe? Ans. 291 dollars.

FIRST OPERATION. Dollars.		SECOND OPERATION. Dollars.	
Minuend	767	Minuend	767
	131		131
	190	Subtrahends	190
	155		155
Subtrahend	476	Remainder	291
Remainder	291		

In the first operation, the *several subtrahends*, having been properly placed, *are added for a single subtrahend*, to be taken from the minuend. In the second, *the subtrahends are subtracted, as they are added, at one operation*, thus: beginning with units, 5 and 1 equal 6, which from 7 units leaves 1 unit; passing to tens, 5 and 9 and 3 equal 17 tens; reserving the left-hand figure to add in with the figures of the subtrahends in the next column, the right-hand figure, 7, being larger than 6 tens of the minuend, we add 10 to the 6, and, subtracting, have left 9 tens; and, passing to hundreds, we add in the left-hand figure 1 reserved from the 17 tens, and also add 1, equal 10 tens, to compensate for the 10 added to the minuend, and with the other figures, 1 and 1 and 1 equal 5 hundreds, which, taken from 7 hundreds, leave 2 hundreds; and 291 as the answer sought.

2. E. Webster owned 6,765 acres of land, and he gave to his oldest brother 2,196 acres, and his uncle Rollins 1,981 acres; how much has he left?

3. The real estate of James Dow is valued at 3,769 dollars, and his personal estate at 2,648 dollars; he owes John Smith 1,728 dollars, and Job Tyler 1,161 dollars; how much is Dow worth?

§ IV. MULTIPLICATION.

MENTAL EXERCISES.

ART. **34.** WHEN any number is to be added to itself several times, the operation may be shortened by a process called *Multiplication.*

Ex. 1. If a man can earn 8 dollars in 1 week, what will he earn in 4 weeks?

ILLUSTRATION. — It is evident, since a man can earn 8 dollars in 1 week, in 4 weeks he will earn 4 times as much, and the result may be obtained by addition; thus, $8+8+8+8=32$; or, by a more convenient process, by setting down the 8 but once, and multiplying it by 4, the number of times it is to be repeated; thus, 4 times 8 are 32. Hence in 4 weeks he will earn 32 dollars.

MULTIPLICATION TABLE.

2 times 1 are 2	3 times 1 are 3	4 times 1 are 4	5 times 1 are 5
2 times 2 are 4	3 times 2 are 6	4 times 2 are 8	5 times 2 are 10
2 times 3 are 6	3 times 3 are 9	4 times 3 are 12	5 times 3 are 15
2 times 4 are 8	3 times 4 are 12	4 times 4 are 16	5 times 4 are 20
2 times 5 are 10	3 times 5 are 15	4 times 5 are 20	5 times 5 are 25
2 times 6 are 12	3 times 6 are 18	4 times 6 are 24	5 times 6 are 30
2 times 7 are 14	3 times 7 are 21	4 times 7 are 28	5 times 7 are 35
2 times 8 are 16	3 times 8 are 24	4 times 8 are 32	5 times 8 are 40
2 times 9 are 18	3 times 9 are 27	4 times 9 are 36	5 times 9 are 45
2 times 10 are 20	3 times 10 are 30	4 times 10 are 40	5 times 10 are 50
2 times 11 are 22	3 times 11 are 33	4 times 11 are 44	5 times 11 are 55
2 times 12 are 24	3 times 12 are 36	4 times 12 are 48	5 times 12 are 60
6 times 1 are 6	7 times 1 are 7	8 times 1 are 8	9 times 1 are 9
6 times 2 are 12	7 times 2 are 14	8 times 2 are 16	9 times 2 are 18
6 times 3 are 18	7 times 3 are 21	8 times 3 are 24	9 times 3 are 27
6 times 4 are 24	7 times 4 are 28	8 times 4 are 32	9 times 4 are 36
6 times 5 are 30	7 times 5 are 35	8 times 5 are 40	9 times 5 are 45
6 times 6 are 36	7 times 6 are 42	8 times 6 are 48	9 times 6 are 54
6 times 7 are 42	7 times 7 are 49	8 times 7 are 56	9 times 7 are 63
6 times 8 are 48	7 times 8 are 56	8 times 8 are 64	9 times 8 are 72
6 times 9 are 54	7 times 9 are 63	8 times 9 are 72	9 times 9 are 81
6 times 10 are 60	7 times 10 are 70	8 times 10 are 80	9 times 10 are 90
6 times 11 are 66	7 times 11 are 77	8 times 11 are 88	9 times 11 are 99
6 times 12 are 72	7 times 12 are 84	8 times 12 are 96	9 times 12 are 108
10 times 1 are 10	10 times 11 are 110	11 times 8 are 88	12 times 4 are 48
10 times 2 are 20	10 times 12 are 120	11 times 9 are 99	12 times 5 are 60
10 times 3 are 30		11 times 10 are 110	12 times 6 are 72
10 times 4 are 40	11 times 1 are 11	11 times 11 are 121	12 times 7 are 84
10 times 5 are 50	11 times 2 are 22	11 times 12 are 132	12 times 8 are 96
10 times 6 are 60	11 times 3 are 33		12 times 9 are 108
10 times 7 are 70	11 times 4 are 44	12 times 1 are 12	12 times 10 are 120
10 times 8 are 80	11 times 5 are 55	12 times 2 are 24	12 times 11 are 132
10 times 9 are 90	11 times 6 are 66	12 times 3 are 36	12 times 12 are 144
10 times 10 are 100	11 times 7 are 77		

QUESTION. — Art. 34. How may the process of adding a number to itself several times be shortened?

2. What cost 5 barrels of flour at 6 dollars per barrel?

ILLUSTRATION. — If a barrel of flour cost 6 dollars, 5 barrels will cost 5 times as much; 5 times 6 are 30. Hence 5 barrels of flour at 6 dollars per barrel will cost 30 dollars.

3. What cost 6 bushels of beans at 2 dollars per bushel?
4. What cost 5 quarts of cherries at 7 cents per quart?
5. What will 7 quarts of vinegar cost at 12 cents per quart?
6. What cost 9 acres of land at 10 dollars per acre?
7. If a pint of currants cost 4 cents, what cost 9 pints?
8. If in 1 penny there are 4 farthings, how many in 9 pence? In 7 pence? In 8 pence? In 4 pence? In 3 pence?
9. If 12 pence make a shilling, how many pence in 3 shillings? In 5 shillings? In 7 shillings? In 9 shillings?
10. If one pound of raisins cost 6 cents, what cost 4 pounds? 5 pounds? 6 pounds? 7 pounds? 8 pounds? 9 pounds? 10 pounds? 12 pounds?
11. In one acre there are four roods; how many roods in 2 acres? In 3 acres? In 4 acres? In 5 acres? In 6 acres? In 9 acres?
12. A good pair of boots is worth 5 dollars; what must I give for 5 pairs? For 6 pairs? For 7 pairs? For 8 pairs? For 9 pairs?
13. A cord of good walnut wood may be obtained for 8 dollars; what must I give for 4 cords? For 6 cords? For 9 cords?
14. What cost 4 quarts of milk at 5 cents a quart, and 8 gallons of vinegar at 10 cents a gallon?
15. If a man earn 7 dollars a week, how much will he earn in 3 weeks? In 4 weeks? In 5 weeks? In 6 weeks? In 7 weeks? In 9 weeks?
16. If 1 thousand feet of boards cost 12 dollars, what cost 4 thousand? 5 thousand? 6 thousand? 7 thousand? 9 thousand? 12 thousand?
17. If 3 pairs of shoes buy 1 pair of boots, how many pairs of shoes will it take to buy 7 pairs of boots?
18. If 5 bushels of apples buy 1 barrel of flour, how many bushels of apples are equal in value to 12 barrels of flour?
19. If 1 yard of canvas cost 25 cents, what will 12 yards cost?

ILLUSTRATION. — The number 25 is composed of 2 tens and 5 units; 12 times 2 tens are 24 tens; and 12 times 5 units are

60 units, or 6 tens; 24 tens added to 6 tens make 30 tens, or 300. Therefore, 12 yards will cost 300 cents, or 3 dollars.

20. In 1 pound there are 20 shillings; how many shillings in 3 pounds? In 4 pounds? In 6 pounds?

21. A gallon of molasses is worth 25 cents; what is the value of 2 gallons? Of 3 gallons? Of 4 gallons? Of 5 gallons? Of 6 gallons? Of 9 gallons?

22. If 1 man earn 12 dollars in 16 days, how much would 10 men earn in the same time?

23. If a steam-engine runs 28 miles in 1 hour, how far will it run in 4 hours? In 6 hours? In 9 hours?

24. If the earth turns on its axis 15 degrees in 1 hour, how far will it turn in 7 hours? In 11 hours? In 12 hours?

25. In a certain regiment there are 8 companies, in each company 6 platoons, and in each platoon 12 soldiers; how many soldiers are there in the regiment?

26. If 1 man walk 7 miles in 1 hour, how far will he walk in 8 hours? In 9 hours? In 11 hours? In 12 hours? In 20 hours? In 30 hours?

ART. **35.** The learner, having performed the foregoing questions, will perceive that

MULTIPLICATION is the process of taking a number as many times as there are units in another number.

In multiplication three terms are employed, called the *Multiplicand*, the *Multiplier*, and the *Product*.

The *multiplicand* is the number to be multiplied or taken.

The *multiplier* is the number by which we multiply, and denotes the number of times the multiplicand is to be taken.

The *product* is the result, or number produced by the multiplication.

The multiplicand and multiplier are often called FACTORS.

SIGNS. — The sign of multiplication is formed by two short lines crossing each other obliquely; thus, $\times$. It shows that the numbers between which it is placed are to be multiplied together; thus, the expression $7 \times 5 = 35$ is read, 7 multiplied by 5 is equal to 35.

QUESTIONS. — Art. 35. What is multiplication? What three terms are employed? What is the multiplicand? What is the multiplier? What is the product? What are the multiplicand and multiplier often called? What is the sign of multiplication? What does it show?

EXERCISES FOR THE SLATE.

ART. **36.** Method of operation when the multiplier does not exceed 12.

Ex. 1. Let it be required to multiply 175 by 7.

Ans. 1225.

	OPERATION.
Multiplicand	175
Multiplier	7
Product	1225

Having written the multiplier under the unit figure of the multiplicand, we multiply the 5 units by 7, obtaining 35, and set down the 5 units directly under the 7, and reserve the 3 tens for the tens' column. We then multiply the 7 tens by 7, obtaining 49, and, adding the 3 tens which were reserved, we have 52 tens, or 5 hundreds and 2 tens. Writing down the 2 tens, and reserving the 5 hundreds, we multiply 1 by 7; and, adding the reserved 5 hundreds, we have 12 hundreds, which we write down in full, and the product is 1225.

EXAMPLES FOR PRACTICE.

	2.	3.	4.
Multiply	8756	4567	7896
By	4	3	5
Ans.	35024	13701	39480

5.	6.	7.	8.
56807	47893	61657	89765
5	6	7	9
284035	287358	431599	807885

9. Multiply 767853 by 9.
10. Multiply 876538765 by 8.
11. Multiply 7654328 by 7.
12. Multiply 4976387 by 5.
13. Multiply 8765448 by 12.
14. Multiply 4567839 by 11.
15. What cost 8675 barrels of flour at 7 dollars per barrel?

QUESTIONS. — Art. 36. How must numbers be written for multiplication? At which hand do you begin to multiply? Why? Where do you write the first or right-hand figure of the product of each figure in the multiplicand? Why? What is done with the tens or left-hand figure of each product? *How, then, do you* proceed when the multiplier does not exceed 12?

16. What cost 25384 tons of hay at 9 dollars per ton?

17. If, on 1 page in this book, there are 2538 letters, how many are there on 11 pages?

ART. 37. Method of operation when the multiplier exceeds 12.

Ex. 1. Let it be required to multiply 763 by 24.

Ans. 18312.

	OPERATION.
Multiplicand	763
Multiplier	24
	3052
	1526
Product	18312

We write the multiplier under the multiplicand, and proceed to multiply the multiplicand by 4, the unit figure of the multiplier, precisely as in Art. 36. We then, in like manner, multiply the multiplicand by the 2 tens in the multiplier, taking care to write the first figure obtained by this multiplication in tens' column, directly under the 2 of the multiplier; and, adding together these *partial* products obtained by the two multiplications, and placed as in the operation, we have the full product of 763 multiplied by 24, which is 18312.

ART. 38. The preceding examples sufficiently illustrate the principle and method of multiplication; hence the following general

RULE. — *Write the multiplier under the multiplicand, arranging units under units, tens under tens, &c.*

Multiply each figure of the multiplicand by each figure of the multiplier, beginning with the right-hand figure, writing the right-hand figure of each product under the figure multiplied, and adding the left-hand figure or figures, if any, to the succeeding product.

If the multiplier consists of more than one figure, the right-hand figure of each partial product must be placed directly under the figure of the multiplier that produces it. The sum of the partial products will be the whole product required.

NOTE. — When there are ciphers between the significant figures of the multiplier, pass over them in the operation, and multiply by the significant figures only, remembering to set the first figure of the product directly under the figure of the multiplier that produces it.

QUESTIONS. — Art. 37. How do you proceed when the multiplier exceeds 12? Where do you set the first figure of each partial product? Why? How is the true product found? — Art. 38. What is the general rule for multiplication? When there are ciphers between the significant figures of the multiplier, how do you proceed?

ART. 39. *First Method of Proof.* — Multiply the multiplier by the multiplicand, and if the result is like the first product, the work is supposed to be right.

The reason of this proof depends on the principle, *That, when two or more numbers are multiplied together, the product is the same, whatever the order of multiplying them.*

Ex. 2. Multiply 7895 by 56.

	OPERATION.	PROOF.
Multiplicand	7895	56
Multiplier	56	7895
	47370	280
	39475	504
Product	442120	448
		392
		Product 442120

NOTE. — The common mode of proof in business is to divide the product by the multiplier, and, if the work is right, the quotient will be like the multiplicand. This mode of proof anticipates the principles of division, and therefore cannot be employed without a previous knowledge of that rule.

ART. 40. *Second Method of Proof.* — Begin at the left hand of the multiplicand, and add together its successive figures toward the right till the sum obtained equals or exceeds the number nine. If it equals it, drop the nine, and begin to add again at this point, and proceed till you obtain a sum equal to, or greater than, nine. If it exceeds nine, drop the nine as before, and carry the excess to the next figure, and then continue the addition as before. Proceed in this way till you have added all the figures in the multiplicand and rejected all the nines contained in it, and write the final excess at the right hand of the multiplicand.

Proceed in the same manner with the multiplier, and write the final excess under that of the multiplicand. Multiply these excesses together, and place the excess of nines in their product at the right.

Then proceed to find the excess of nines in the product obtained by the original operation; and, if the work is right,

QUESTIONS. — Art. 39. How is multiplication proved by the first method? What is the reason for this method of proof? What is the common mode of proof in business? — Art. 40. What is the second method of proving multiplication?

the excess thus found will be equal to the excess contained in the product of the above excesses of the multiplicand and multiplier.

Ex. 3.

OPERATION.

Multiplicand	12345 =	6 excess.	
Multiplier	2231 =	8 excess.	
	12345	48 = 3	
	37035		
	24690		Proof.
	24690		
Product	27541695 =	3	

NOTE. — This method of proof, though perhaps sufficiently sure for common purposes, is not always a test of the correctness of an operation. If two or more figures in the work should be transposed, or the value of one figure be just as much too great as another is too small, or if a nine be set down in the place of a cipher, or the contrary, the excess of nines will be the same, and still the work may not be correct. Such a balance of errors will not, however, be likely to occur.

EXAMPLES FOR PRACTICE.

	4.	5.
Multiply	67895	78956
By	36	47
	407370	552692
	203685	315824
Ans.	2444220	3710932

	6.	7.
Multiply	89325	47896
By	901	2008
	89325	383168
	803925	95792
Ans.	80481825	96175168

8. What cost 47 hogsheads of molasses at 13 dollars per hogshead?

9. What cost 97 oxen at 29 dollars each?

QUESTIONS. — Is this method of proof always a true test of the correctness of an operation? What is the reason for this method of proof?

10. Sold a farm containing 367 acres, at 97 dollars per acre what was the amount?

11. An army of 17006 men receive each 109 dollars as their annual pay; what is the amount paid the whole army?

12. If a mechanic deposit annually in the Savings Bank 407 dollars, what will be the sum deposited in 27 years?

13. If a man travel 37 miles in 1 day, how far will he travel in 365 days?

14. If there be 24 hours in 1 day, how many hours in 365 days?

15. How many gallons in 87 hogsheads, there being 63 gallons in each?

16. If the expenses of the Massachusetts Legislature be 1839 dollars per day, what will be the amount in a session of 109 days?

17. If a hogshead of sugar contains 368 pounds, how many pounds in 187 hogsheads?

18. Multiply 675 by 476.

19. Multiply 679 by 763.

20. Multiply 899 by 981.

21. Multiply 7854 by 1234.

22. Multiply 3001 by 6071.

23. Multiply 7117 by 9876.

24. Multiply 376546 by 407091.

25. Multiply 7001009 by 7007867.

26. Multiply five hundred and eighty-six by nine hundred and eight.

27. Multiply three thousand eight hundred and five by one thousand and seven.

28. Multiply two thousand and seventy-one by seven hundred and six.

29. Multiply eighty-eight thousand and eight by three thousand and seven.

30. Multiply ninety thousand eight hundred and seven by one thousand and ninety-one.

31. Multiply ninety thousand eight hundred and seven by nine thousand one hundred and six.

32. Multiply fifty thousand and one by five thousand eight hundred and seven.

33. Multiply eighty thousand and nine by nine thousand and sixteen.

34. Multiply forty-seven thousand and thirteen by eighty thousand eight hundred and seven.

ART. 41. A COMPOSITE number is one produced by multiplying together two or more numbers greater than unity or one; thus, 12 is a composite number, since it is the product of 3×4; and also 24 is a composite number, since it is the product of $2 \times 3 \times 4$.

A FACTOR of any number is a name given to one of two or more numbers, which, being multiplied together, produce that number; thus, 3 and 4 are factors of 12, since $3 \times 4 = 12$.

ART. 42. To multiply by a composite number.

EX. 1. A merchant bought fifteen pieces of broadcloth, at 96 dollars per piece; how much did he pay for the whole?

Ans. 1440 dollars.

OPERATION.

96 dolls., price of 1 piece.
3
288 dolls., price of 3 pieces.
5
1440 dolls., price of 15 pieces.

The factors of 15 are 3 and 5. Now, if we multiply the price of one piece by the factor 3, we get the cost of 3 pieces; and then, by multiplying the cost of 3 pieces by the factor 5, it is evident we obtain the cost of 15, the number of pieces bought, since 15 is equal to 5 times 3. Hence we adopt the following

RULE. — *Multiply the multiplicand by one of the factors of the multiplier, and the product thus obtained by another, and so on until each of the factors has been used as a multiplier. The last product will be the answer.*

NOTE. — The product of any number of factors is the same in whatever order they are multiplied. Thus, $3 \times 4 = 12$, and $4 \times 3 = 12$.

EXAMPLES FOR PRACTICE.

2. Multiply 30613 by $25 = 5 \times 5$. Ans. 765325.
3. Multiply 1469 by $84 = 7 \times 12$. Ans. 123396.
4. Multiply 7546 by 81, using its factors.
5. Multiply 7901 by 125, using its factors.
6. In 1 mile there are 63360 inches; how many inches in 45 miles?
7. If in one year there are 8766 hours, how many hours in 72 years?

QUESTIONS. — Art. 41. What is a composite number? What is a factor of any number? — Art. 42. What are the factors of 15? How do we multiply by a composite number? Repeat the rule. In what order must the factors of a composite number be multiplied?

8. If sound moves 1142 feet in a second, how far will it move in one minute?

ART. 43. When the multiplier is 1 with one or more ciphers annexed to it, as 10, 100, &c.

Ex. 1. In 1 day there are 24 hours; how many hours in 10 days? In 100 days? Answers. 240, 2400.

OPERATION.

Multiplicand	24	24
Multiplier	10	100
Product	240	2400

Or thus, 240, 2400.

The removal of a figure one place to the left increases its value *ten times*. (Art. 7.) If, then, we annex a cipher to the right of 24, the multiplicand, each figure is removed one place to the left, and its value is increased ten times, or multiplied by 10. If two ciphers are annexed, each figure is removed two places to the left, and its value is increased 100 times, or multiplied by 100; every additional cipher increasing the value *ten times*. Hence the following

RULE. — *Annex to the multiplicand as many ciphers as has the multiplier. The number thus formed will be the product required.*

EXAMPLES FOR PRACTICE.

2. Multiply 2356 by 10.
3. Multiply 5873 by 100.
4. Multiply 7964 by 1000.
5. Multiply 98725 by 100000.

ART. 44. When there are ciphers on the right hand of the multiplier or multiplicand, or both.

Ex. 1. What will 600 acres of land cost at 20 dollars per acre? Ans 12,000 dollars.

OPERATION.

Multiplicand	600
Multiplier	20
Product	12000

The multiplicand may be resolved into the factors 6 and 100, and the multiplier into the factors 2 and 10. Now, it is evident (Art. 42), if these several factors be multiplied together, they will produce the same product as the original factors 600 and 20. Thus $6 \times 2 = 12$, and $12 \times 100 = 1200$, and $1200 \times 10 = 12000$, the same result as in the operation. Hence the following

QUESTIONS. — Art. 43. What is the effect of removing a figure one place to the left? What is the effect of annexing a cipher to any figure or number? Two ciphers? &c. What is the rule when the multiplier is 1 with ciphers annexed? — Art 44. How do you arrange the figures for multiplication, when there are ciphers on the right hand of either the multiplier or multiplicand, or both? Why does multiplying the significant figures and annexing the ciphers produce the true product?

RULE. — *Write the significant figures of the multiplier under those of the multiplicand, and multiply them together. To their product annex as many ciphers as there are on the right of both multiplicand and multiplier.*

EXAMPLES FOR PRACTICE.

	2.	3.
Multiply	8785324	713378900
By	3200	70080
	17570648	57070312
	26355972	49936523
Ans.	28113036800	49993593312000

4. Multiply 8010700 by 9000909.

Ans. 72103581726300.

5. Multiply 700110000 by 700110000.

6. Multiply 4070607 by 7007000.

7. Multiply 4110000 by 1017010.

8. Multiply twenty-nine millions two thousand nine hundred and nine by four hundred and four thousand.

9. Multiply eighty-seven millions by eight hundred thousand seven hundred.

10. Multiply one million one thousand one hundred by nine hundred nine thousand and ninety.

11. Multiply forty-nine millions and forty-nine by four hundred and ninety thousand.

12. Multiply two hundred millions two hundred by two millions two thousand and two.

13. Multiply four millions forty thousand four hundred by three hundred three thousand.

14. Multiply three hundred thousand thirty by forty-seven thousand seventy.

15. Multiply fifteen millions one hundred by two thousand two hundred.

16. Multiply one billion twenty thousand by one thousand one hundred.

QUESTION. — What is the rule?

§ V. DIVISION.

MENTAL EXERCISES.

ART. 45. WHEN it is required to find how many times one number contains another, the process is called *Division.*

EX. 1. A boy has 32 cents, which he wishes to give to 8 of his companions, to each an equal number; how many must each receive?

ILLUSTRATION. — It is evident that each boy must receive as many cents as the number 8 is contained times in 32. We therefore inquire what number 8 must be multiplied by to make 32. By trial, we find that 4 is the number; because 4 times 8 make 32. Hence 8 is contained in 32 4 times, and the boys receive 4 cents apiece.

The following table should be studied by the learner to aid him in solving questions in division:

DIVISION TABLE.

2 in	2	1 time	3 in	3	1 time	4 in	4	1 time	5 in	5	1 time
2 in	4	2 times	3 in	6	2 times	4 in	8	2 times	5 in	10	2 times
2 in	6	3 times	3 in	9	3 times	4 in	12	3 times	5 in	15	3 times
2 in	8	4 times	3 in	12	4 times	4 in	16	4 times	5 in	20	4 times
2 in	10	5 times	3 in	15	5 times	4 in	20	5 times	5 in	25	5 times
2 in	12	6 times	3 in	18	6 times	4 in	24	6 times	5 in	30	6 times
2 in	14	7 times	3 in	21	7 times	4 in	28	7 times	5 in	35	7 times
2 in	16	8 times	3 in	24	8 times	4 in	32	8 times	5 in	40	8 times
2 in	18	9 times	3 in	27	9 times	4 in	36	9 times	5 in	45	9 times
2 in	20	10 times	3 in	30	10 times	4 in	40	10 times	5 in	50	10 times
2 in	22	11 times	3 in	33	11 times	4 in	44	11 times	5 in	55	11 times
2 in	24	12 times	3 in	36	12 times	4 in	48	12 times	5 in	60	12 times
6 in	6	1 time	7 in	7	1 time	8 in	8	1 time	9 in	9	1 time
6 in	12	2 times	7 in	14	2 times	8 in	16	2 times	9 in	18	2 times
6 in	18	3 times	7 in	21	3 times	8 in	24	3 times	9 in	27	3 times
6 in	24	4 times	7 in	28	4 times	8 in	32	4 times	9 in	36	4 times
6 in	30	5 times	7 in	35	5 times	8 in	40	5 times	9 in	45	5 times
6 in	36	6 times	7 in	42	6 times	8 in	48	6 times	9 in	54	6 times
6 in	42	7 times	7 in	49	7 times	8 in	56	7 times	9 in	63	7 times
6 in	48	8 times	7 in	56	8 times	8 in	64	8 times	9 in	72	8 times
6 in	54	9 times	7 in	63	9 times	8 in	72	9 times	9 in	81	9 times
6 in	60	10 times	7 in	70	10 times	8 in	80	10 times	9 in	90	10 times
6 in	66	11 times	7 in	77	11 times	8 in	88	11 times	9 in	99	11 times
6 in	72	12 times	7 in	84	12 times	8 in	96	12 times	9 in	108	12 times
10 in	10	1 time	10 in	110	11 times	11 in	88	8 times	12 in	48	4 times
10 in	20	2 times	10 in	120	12 times	11 in	99	9 times	12 in	60	5 times
10 in	30	3 times				11 in	110	10 times	12 in	72	6 times
10 in	40	4 times	11 in	11	1 time	11 in	121	11 times	12 in	84	7 times
10 in	50	5 times	11 in	22	2 times	11 in	132	12 times	12 in	96	8 times
10 in	60	6 times	11 in	33	3 times				12 in	108	9 times
10 in	70	7 times	11 in	44	4 times	12 in	12	1 time	12 in	120	10 times
10 in	80	8 times	11 in	55	5 times	12 in	24	2 times	12 in	132	11 times
10 in	90	9 times	11 in	66	6 times	12 in	36	3 times	12 in	144	12 times
10 in	100	10 times	11 in	77	7 times						

2. A farmer received 8 dollars for 2 sheep; what was the price of each?

ILLUSTRATION. — It is evident, since he received 8 dollars for 2 sheep, for 1 sheep he must receive as many dollars as 2 is contained times in 8. 2 is contained in 8 4 times, because 4 times 2 are 8; hence 4 dollars was the price of each sheep.

3. A man gave 15 dollars for 3 barrels of flour; what was the cost of each barrel?

4. A lady divided 20 oranges among her 5 daughters; how many did each receive?

5. If 4 casks of lime cost 12 dollars, what costs 1 cask?

6. A laborer earned 48 shillings in 6 days; what did he receive per day?

7. A man can perform a certain piece of labor in 30 days; how long will it take five men to do the same?

8. When 72 dollars are paid for 8 acres of land, what costs 1 acre? What cost 3 acres?

9. If 21 pounds of flour can be obtained for 3 dollars, how much can be obtained for 1 dollar? How much for 8 dollars? How much for 9 dollars?

10. Gave 56 cents for 8 pounds of raisins; what costs 1 pound? What cost 7 pounds?

11. If a man walk 24 miles in 6 hours, how far will he walk in 1 hour? How far in 10 hours?

12. Paid 56 dollars for 7 hundred weight of sugar; what costs 1 hundred weight? What cost 10 hundred weight?

13. If 5 horses will eat a load of hay in 1 week, how long would it last 1 horse?

14. In 20, how many times 2? How many times 4? How many times 5? How many times 10?

15. In 24, how many times 3? How many times 4? How many times 6? How many times 8?

16. How many times 7 in 21? In 28? In 56? In 35? In 14? In 63? In 77? In 70? In 84?

17. How many times 6 in 12? In 36? In 18? In 54? In 60? In 42? In 48? In 72? In 66?

18. How many times 9 in 27? In 45? In 63? In 81? In 99? In 108?

19. How many times 11 in 22? In 55? In 77? In 88? In 110? In 132?

20. How many times 12 in 36? In 60? In 72? In 84? In 120? In 144?

ART. 46. The pupil will now perceive that

DIVISION is the process of finding how many times one number is contained in another.

In division there are three principal terms: the *Dividend*, the *Divisor*, and the *Quotient*, or *answer*.

The *dividend* is the number to be divided.

The *divisor* is the number by which we divide.

The *quotient* is the number of times the divisor is contained in the dividend.

NOTE. — *Quotient* is derived from the Latin word *quoties*, which signifies *how often*, or *how many times*.

When the dividend does not contain the divisor an exact number of times, the *excess* is called a *remainder*, and may be regarded as a *fourth* term in the *division*. The remainder, being part of the dividend, will always be of the same denomination or kind as the dividend, and must always be less than the divisor.

ART. 47. SIGNS. — The sign of division is a short horizontal line, with a dot above it and another below; thus, $\div$. It shows that the number *before* it is to be divided by the number *after* it. The expression $6 \div 2 = 3$ is read, 6 divided by 2 is equal to 3.

Division is also indicated by writing the dividend above a short horizontal line and the divisor below; thus, $\frac{6}{2}$. The expression $\frac{6}{2} = 3$ is read, 6 divided by 2 is equal to 3.

There is a third method of indicating division, by a curved line placed between the divisor and dividend. Thus, the expression $6\,)\,12$ shows that 12 is to be divided by 6.

EXERCISES FOR THE SLATE.

ART. 48. The method of operation by *Short Division*, or when the divisor does not exceed 12.

Ex. 1. Divide 8574 dollars equally among 6 men.

Ans. 1429 dollars.

QUESTIONS. — Art. 46. What is division? What are the three principal terms in division? What is the dividend? What is the divisor? What is the quotient? What the remainder? What will be the denomination of the remainder? How does it compare with the divisor? — Art. 47. What is the first sign of division, and what does it show? What is the second, and what does it show? What is the third, and what does it show? — Art. 48. What is short division?

OPERATION.

Divisor 6) 8574 Dividend.

1429 Quotient.

We first inquire how many times 6, the divisor, is contained in 8, the first figure of the dividend, which is thousands, and find it to be 1 time, and 2 thousands remaining. We write the 1 directly under the 8, its dividend, for the thousands' figure of the quotient. To 5, the next figure of the dividend, which is hundreds, we regard as prefixed the 2 thousands remaining, which equal 20 hundreds, and thus form the number 25 hundreds, in which we find the divisor 6 to be contained 4 times, and 1 hundred remaining. We write the 4 for the hundreds' figure in the quotient, and the 1 hundred remaining, equal 10 tens, we regard as prefixed to 7, the next figure of the dividend, which is tens, forming 17 tens, in which the divisor 6 is contained 2 times, and 5 tens remaining. We write the 2 for the tens' figure in the quotient, and the 5 tens remaining, equal 50 units, we regard as prefixed to 4, the last figure of the dividend, which is units, forming 54 units, in which the divisor 6 is contained 9 times. Writing the 9 for the units' figure of the quotient, we have 1429 as the entire quotient, or the number of times which the dividend contains the divisor 6.

ART. **49.** From the foregoing illustration we deduce the following

RULE. — *Write the divisor at the left hand of the dividend, with a curved line between them, and draw a horizontal line under the dividend.*

Then, beginning at the left, find how many times the divisor is contained in the fewest figures of the dividend that will contain it, and write the quotient under its dividend.

If there be a remainder, regard it as prefixed to the next figure of the dividend, and divide as before.

Should any dividend be less than the divisor, write a cipher in the quotient, and annex another figure, if any remains, for a new dividend.

NOTE 1. — When there is a remainder after dividing the last figure of the dividend, write it with the divisor underneath, with a line between them, at the right of the quotient.

NOTE 2. — *Prefix* means to place before; *annex*, to place after.

ART. **50.** *First Method of Proof.* — Multiply the divisor by the quotient, and to the product add the remainder, if any, and, if the work is right, the sum thus obtained will be equal to the dividend.

QUESTIONS. — How are the numbers arranged for short division? At which hand do you begin to divide? Why not begin at the right, where you begin to multiply? Where do you write the quotient? If there is a remainder after dividing a figure, what is done with it? — Art. 49. What is the rule for short division? Repeat the notes.

NOTE. — It will be seen, from this method of proof, that division is the reverse of multiplication. The *dividend* answers to the *product*, the *divisor* to one of the *factors*, and the *quotient* to the *other*.

EXAMPLES FOR PRACTICE.

2. Divide 6375 by 5.

OPERATION.

Divisor 5) 6 3 7 5 Dividend.

1 2 7 5 Quotient.

PROOF.

1 2 7 5 Quotient.
5 Divisor.

6 3 7 5 Dividend.

3. 3) 7 8 9 3 7 6 2

2 6 3 1 2 5 4

4. 4) 4 7 6 3 2 5 6

1 1 9 0 8 1 4

5. 5) 3 7 8 9 5 6 5

6. 6) 8 7 6 5 3 8 9

7. 7) 9 8 7 6 3 5

8. 8) 3 7 8 5 3 2

9. 9) 8 9 5 3 7 8 4

10. 11) 7 6 7 8 9 0 3

11. 12) 6 3 4 5 3 2 1

		Quotients.
12.	Divide 479956 by 6.	$79992\frac{4}{6}$
13.	Divide 385678 by 7.	$55096\frac{6}{7}$
14.	Divide 438789 by 8.	$54848\frac{5}{8}$
15.	Divide 1678767 by 9.	$186529\frac{6}{9}$
16.	Divide 11497583 by 12.	$958131\frac{11}{12}$

		Quotients.	Rem.
17.	Divide 5678956 by 5.		1
18.	Divide 1135791 by 7.		6
19.	Divide 1622550 by 8.		6
20.	Divide 2028180 by 9.		3
21.	Divide 2253530 by 12.		2
22.	Divide 1877940 by 11.		9
	Sum of the quotients,	2084732	27

QUESTIONS. — Art. 50. How is short division proved? Of what is division the reverse? To what do the three terms in division answer in multiplication? What, then, is the reason for this proof of division?

23. Divide 944,580 dollars equally among 12 men, and what will be the share of each?

24. Divide 154,503 acres of land equally among 9 persons.

25. A plantation in Cuba was sold for 7,011,608 dollars, and the amount was divided among 8 persons. What was paid to each person?

26. A prize, valued at 178,656 dollars, is to be equally divided among 12 men; what is the share of each?

27. Among 7 men, 67,123 bushels of wheat are to be distributed; how many bushels does each man receive?

28. If 9 square feet make 1 square yard, how many yards in 895,347 square feet?

29. A township of 876,136 acres is to be divided among 8 persons; how many acres will be the portion of each?

30. Bought a farm for 5670 dollars, and sold it for 7896 dollars, and I divide the net gain among 6 persons; what does each receive?

31. If 6 shillings make a dollar, how many dollars in 7890 shillings?

ART. 51. The method of operation by *Long Division*, or, in general, when the divisor exceeds 12.

EX. 1. A gentleman divided 896 dollars equally among his 7 children; how much did each receive? Ans. 128 dollars.

OPERATION.

Dividend.

Divisor 7) 8 9 6 (1 2 8 Quotient.
7
---
1 9
1 4
---
5 6
5 6

Having set down the divisor and dividend as in short division, we draw a curved line at the right of the dividend, to mark the place for the quotient. We then inquire how many times 7, the divisor, is contained in the 8 hundreds of the dividend; and, finding it to be 1 hundred times, we write the 1 in the quotient,

QUESTIONS. — Art. 51. What is long division? What is the difference between long division and short division? How do you arrange the numbers for long division? What do you first do after arranging the numbers for long division? Where do you place the figures of the quotient?

and multiply the divisor, 7, by it, writing the product, 7 hundreds, under the 8 hundreds, from which we subtract it, and to the remainder, 1, annex the 9 tens of the dividend, making 19 tens. We now inquire how many times 7 is contained in 19 tens, and write the number, 2, at the right of the quotient figure before obtained. We then multiply the divisor by it, and place the product under the 19, and subtract as before; and to the remainder, 5, we annex 6 units, the next and last figure of the dividend, making 56 units. We proceed, as before, to find the next quotient figure, and, after subtracting the product of the divisor multiplied by it from 56, find there is no remainder left. Hence we learn that each one of the 7 children must receive 128 dollars.

NOTE. — The preceding example and the four that follow are usually performed by short division, but are here introduced to illustrate more clearly the method of operation by long division.

EXAMPLES FOR PRACTICE.

2. Divide 1728 by 8.
3. Divide 987656 by 11.
4. Divide 123456789 by 9.
5. Divide 390413609 by 12.

Ex. 6. A gentleman divided 4712 dollars equally among his 19 sons; what was the share of each? Ans. 248 dollars.

OPERATION.

Dividend.

Divisor 19)4712(248 Quotient.
38

91
76

152
152

We first inquire how many times 19, the divisor, is contained in 47, the two left-hand figures of the dividend; and, finding it to be 2 times, we write the 2 in the quotient, multiply the divisor by it, and subtract the product from the 47; and to the remainder, 9, annex 1, the next figure of the dividend, making 91. We next inquire how many times 19 is contained in 91, place the number, 4, in the quotient, then multiply and subtract as before, and to the remainder, 15, annex 2, the last figure of the dividend, and, proceeding as before, after finding the quotient figure, no remainder is left. Hence the share of each of the 19 sons is 248 dollars. This illustration, except in omissions, is essentially like the preceding one.

QUESTIONS. — After the quotient figure is found, what is the next thing you do? Where do you place the product? What do you next do? What is the next step? How do you then proceed? Is long division the same in principle as short division?

ART. **52.** From the preceding illustrations, the pupil will perceive the propriety of the following general

RULE. — *Write the divisor and dividend as in short division, and draw a curved line at the right hand of the dividend.*

Then inquire how many times the divisor is contained in the fewest figures on the left hand of the dividend that will contain it, and write the result at the right hand of the dividend for the first quotient figure.

Multiply the divisor by the quotient figure, and subtract the product from the figures of the dividend used, and to the remainder annex the next figure of the dividend.

Find how many times the divisor is contained in the number thus formed; write the figure denoting it at the right hand of the former quotient figure.

Thus proceed until all the figures of the dividend are divided.

NOTE 1. — The proper remainder is in all cases *less* than the divisor. If, in the course of the operation, it is at any time found to be as large as, or *larger* than, the divisor, it will show that there is an error in the work, and that the quotient figure should be increased.

NOTE 2. — If, at any time, the divisor, multiplied by the quotient figure, produces a product *larger* than the part of the dividend used, it shows that the quotient figure is too *large*, and must be diminished.

NOTE 3. — It will often happen that, when a figure is brought down, the number will not contain the divisor; and in that case a cipher must be placed in the quotient, and another figure of the dividend brought down, and so on until the number is large enough to contain the divisor.

NOTE 4. — If there is a remainder after dividing all the figures of the dividend, it must be written as directed in the preceding rule.

ART. **53.** *Second Method of Proof.* — Add together the remainder, if any, and all the products that have been produced by multiplying the divisor by the several quotient figures, and the result will be like the dividend, if the work is right.

ART. **54.** *Third Method.* — Subtract the remainder, if any, from the dividend, and divide the difference by the quotient. The result will be like the original divisor, if the work is right.

NOTE. — The first method of proof (Art. 50) is usually most convenient, and is most commonly employed.

QUESTIONS. — Art. 52. What is the general rule for long division? How may you know when the quotient figure is too small? How may you know when it is too large? What do you do when the part of the dividend used will not contain the divisor? — Art. 53. What is the second method of proof for division? — Art. 54. What is the third method? Can long division be proved by the first method of proof (Art. 50)?

EXAMPLES FOR PRACTICE.

Ex. 7. It is required to find how many times 48 is contained in 28618. Ans. 596.

OPERATION.

Dividend.
Divisor 48) 28618 (596 Quotient.
240
461
432
298
288
10 Remainder.

PROOF BY MULTIPLICATION

596 Quotient.
48 Divisor.
4768
2384
28608
10 Remainder.
28618 Dividend.

8.

OPERATION.

Dividend.
Divisor 26) 5698 (219 Quotient.
*+52
49
+26
238
+234
+4 Remainder.

PROOF BY ADDITION.

52
26 } Products.
234
4 Remainder.
5698 Dividend.

9.

OPERATION.

Dividend.
Divisor 144) 13824 (96 Quotient.
1296
864
864

PROOF BY DIVISION.

Dividend.
96) 13824 (144 Divisor
96
422
384
384
384

	Quotients.	Rem
10. Divide 3276 by 14.	234	
11. Divide 6205 by 17.	365	

* This sign of addition denotes the several products to be added.

	Quotients.	Rem.
12. Divide 3051 by 21.	145	6
13. Divide 190850 by 25.	7634	0
14. Divide 218579 by 42.		11
15. Divide 9012345 by 31.		25
16. Divide 6717890 by 98.		88
17. Divide 4567890 by 19.		5
18. Divide 1357901 by 87.		5
19. Divide 9988891 by 77.		66
20. Divide 9999999 by 69.		36
21. Divide 867532 by 59.		55
22. Divide 167008 by 87.		55
23. Divide 345678 by 379.		30
24. Divide 3456789567 by 987.		714
25. Divide 8997744444 by 345.		234
26. Divide 4500700701 by 407.		277
27. Divide 6789563 by 1234.		95
28. Divide 78112345 by 8007.		4060
29. Divide 34533669 by 9999.		7122
30. Divide 99999999 by 3333.		0
31. Divide 47856712 by 1789.		962
32. Divide 345678901765 by 4007.		480
33. Divide 478656785178 by 56789.		22346
34. Divide 678957000107 by 10789561.		2295060
35. Divide 990070171009 by 900700601.		200210510

36. Divide three hundred twenty-one thousand three hundred dollars equally among six hundred seventy-five men.

37. Four hundred seventy-one men purchase a township containing one hundred eighty-six thousand forty-five acres; what is the share of each?

38. A railroad, which cost five hundred eighteen thousand seventy-seven dollars, is divided into six hundred seventy-nine shares; what is the value of each share?

39. Divide forty-two thousand four hundred thirty-five bushels of wheat equally among one hundred twenty-three men.

40. A prize, valued at one hundred eighty-four thousand seven hundred seventy-five dollars, is to be divided equally among four hundred seventy-five men; what is the share of each?

41. A certain company purchased a valuable township for nine millions six hundred ninety-one thousand eight hundred

thirty-six dollars; each share was valued at seven thousand eight hundred fifty-four dollars; of how many men did the company consist?

42. A tax of thirty millions fifty-six thousand four hundred sixty-five dollars is assessed equally on four thousand five hundred ninety-seven towns; what sum must each town pay?

Art. **55.** Method of operation when the divisor is a composite number.

Ex. 1. A merchant bought 15 pieces of broadcloth for 1440 dollars; what was the value of each piece? Ans. 96 dollars.

OPERATION.

3) 1 4 4 0 dolls., cost of 15 pieces.

5) 4 8 0 dolls., cost of 5 pieces.

9 6 dolls., cost of 1 piece.

The factors of 15 are 3 and 5. Now, if we divide the 1440 dollars, the cost of 15 pieces, by 3, we obtain 480 dollars, which is evidently the cost of 5 pieces, because there are 5 times 3 in 15. Then, dividing 480 dollars, the cost of 5 pieces, by 5, we get the cost of 1 piece. Hence we deduce the following

Rule. — *Divide the dividend by one of the factors, and the quotient thus found by another, and thus proceed till every factor has been made a divisor. The last quotient will be the true quotient required.*

Examples for Practice.

	Quotients.
2. Divide 765325 by 25 = 5 × 5.	30613
3. Divide 123396 by 84 = 7 × 12.	1469
4. Divide 611226 by 81, using its factors.	
5. Divide 987625 by 125, using its factors.	
6. Divide 17472 by 96, using its factors.	
7. Divide 34848 by 132, using its factors.	

Art. **56.** Method of finding the true remainder when there are several in the operation.

Ex. 1. How many months of 4 weeks each are there in 298 days, and how many days remaining?

Ans. 10 months and 18 days.

Questions. — Art. 55. What are the factors of 15? What do you get the cost of, in this example, when you divide by the factor 3? What, when you divide by 5? Why? What is the rule for dividing by a composite number?

OPERATION.

7) 2 9 8

4) 4 2, 4 days } 18 days.

1 0, 2 weeks }

Since there are 7 days in 1 week, we first divide the 298 days by 7, and have 42 weeks and a remainder of 4 days. Then, since 4 weeks make 1 month, we divide the 42 weeks by 4, and have 10 months and a remainder of 2 weeks. Now, to find the true remainder in days, it is evident that we must multiply the 2 weeks by 7, because 7 days make a week, and to the product add the 4 days; thus, $2 \times 7 = 14$, and $14 + 4 = 18$ days, for the remainder. Hence the following

RULE. — *Multiply each remainder, except the first, by all the divisors preceding the one which produced it; and the first remainder being added to the sum of the products, the amount will be the true remainder.*

NOTE. — There will be but one product to add to the first remainder when there are only two divisors and two remainders.

EX. 2. Divide 789 by 36, using the factors 2, 3, and 6, and find the true remainder. Ans. 33.

OPERATION.

2) 7 8 9

3) 3 9 4, 1, 1st Rem.

6) 1 3 1, 1, 2d Rem.

2 1, 5, 3d Rem.

FINDING THE TRUE REMAINDER.

$5 \times 3 \times 2 = 30$, 1st Product.

$1 \times 2 = 2$, 2d Product.

1, 1st Remainder.

33, true Rem.

EXAMPLES FOR PRACTICE.

3. Divide 934 by 55, using the factors 5 and 11, and find the true remainder. Ans. 54.

4. Divide 5348 by 48, using the factors 6 and 8, and find the true remainder.

5. Divide 5873 by 84, using the factors 3, 4, and 7, and find the true remainder.

6. Divide 249237 by 1728, using the factors 12, 6, 6, and 4, and find the true remainder.

ART. **57.** When the divisor is 1, with one or more ciphers at the right; as 10, 100, &c.

EX. 1. Divide 356 dollars equally among 10 men; what will each man have? Ans. $35\frac{6}{10}$ dollars.

QUESTIONS. — Art. 56. When there are several remainders, what is the rule for finding the true remainder? Will you give the reason for this rule?

OPERATION.
1 | 0) 3 5 | 6

Quotient 3 5, 6 Rem.
Or thus, 3 5 | 6.

It will be remembered, that to multiply by 10 we annex one cipher, which removes the figures one place to the left, and thus *increases* their value *ten times*. Now, it is obvious that if we *reverse* the process, and cut off the right-hand figure by a line, we remove the remaining figures *one* place to the *right*, and consequently *diminish* the value of each *ten times*, and thus divide the whole number by 10. The figures on the left of the line are the quotient, and the one on the right is the remainder, which may be written over the divisor, and annexed to the quotient. Hence the share of each man is $35\frac{6}{10}$ dollars.

EXAMPLES FOR PRACTICE.

	Quotient.	Rem.
2. Divide 6892 by 10.	689	2
3. Divide 4375 by 100.		
4. Divide 24815 by 1000.		
5. Divide 987654321123 by 100000000.		

ART. **58.** When the divisor has ciphers on the right, and is not 10, 100, &c.

Ex. 1. If I divide 5832 pounds of bread equally among 600 soldiers, what is each one's share? Ans. $9\frac{432}{600}$ pounds.

OPERATION.
1 | 0 0) 5 8 | 3 2

6) 5 8, 3 2, 1st Rem.
9, 4, 2d Rem.

Or thus, 6 | 0 0) 5 8 | 3 2
9, 4 3 2

The divisor, 600, may be resolved into the factors 6 and 100. We first divide by the factor 100, by cutting off two figures at the right, and get 58 for the quotient and 32 for a remainder. We then divide the quotient, 58, by the other factor, 6, and obtain 9 for the quotient and 4 for a remainder. The last remainder, 4, being multiplied by the divisor, 100, and 32, the first remainder, added, we obtain 432 for the true remainder (Art. 56). Hence each soldier receives $9\frac{432}{600}$ pounds.

ART. **59.** From the preceding illustrations is deduced the general

RULE. — *Cut off the ciphers from the divisor, and the same number of figures from the right of the dividend.*

Then divide the remaining figures of the dividend by the remaining figures of the divisor.

QUESTIONS. — Art. 57. How do you divide by 10? How does it appear that this divides the number by 10? — Art. 58. How do you divide by 600 *in the example*? How does it appear that this divides the number? — Art. 59. *What is the* general rule?

NOTE. — When by the operation there is a last remainder, to it must be annexed the figures cut off from the dividend to form the true remainder. Should there be no last remainder, then the significant figures, if any, cut off from the dividend, will form the true remainder.

EXAMPLES FOR PRACTICE.

	Quotients.	Rem.
2. Divide 3594 by 80.	44	74
3. Divide 79872 by 240.	332	
4. Divide 467153 by 700.	667	
5. Divide 13112297 by 8900.		
6. Divide 71897654325 by 700000000.		
7. Divide 3456789123456787 by 990000.		
8. Divide 967231731328000 by 1020000000.		
9. Divide 33166405115000 by 1600000000.		
10. Divide 18191618562300 by 1000000000.		
11. Divide 4766666000000 by 55550000000.		

§ VI. QUESTIONS INVOLVING FRACTIONS.

ART. **60.** IF a unit or individual thing is divided into equal parts, each of the parts is called a *Fraction* of the number or thing divided. *Hence* a FRACTION *is one or more equal parts of a unit.*

ILLUSTRATIONS. — 1. When any number or thing is divided into *two* equal parts, *one* of the parts is called *one half*, and is written thus: $\frac{1}{2}$.

2. When any number or thing is divided into *three* equal parts, *one* of the parts is called *one third* ($\frac{1}{3}$); *two* of the parts are called *two thirds* ($\frac{2}{3}$).

3. When any number or thing is divided into *four* equal parts, *one* of the parts is called *one fourth* ($\frac{1}{4}$); *three* of the parts, *three fourths* ($\frac{3}{4}$).

4. When any number or thing is divided into *five* equal parts, *one* of the parts is called *one fifth* ($\frac{1}{5}$); *two* parts, *two fifths* ($\frac{2}{5}$); *three* parts, *three fifths* ($\frac{3}{5}$); and *four* parts, *four fifths* ($\frac{4}{5}$).

QUESTIONS. — Art. 60. What is a fraction? What is meant by one half of any number or thing? How is it written? What is meant by one third, and how is it written? What by one fourth, and how written? What by one fifth, and how written? What by four fifths, and how written? How do you find one half of any number? How one third? How one fourth? &c. How many halves make a whole one? How many thirds? How many fourths? How many fifths?

5. When any number or thing is divided into *six* equal parts, what is *one* of the parts called? Two parts? Five parts?

6. When a number or thing is divided into 7 equal parts, what is 1 part called? 2 parts? 3 parts? 4 parts? 5 parts? 6 parts?

7. When a number or thing is divided into 9 equal parts, what is 1 part called? 2 parts? 4 parts? 5 parts? 7 parts? 8 parts?

8. What is 1 *half* of 4? Of 8? Of 16? Of 20? Of 28? Of 32?

9. What is 1 *third* of 9? Of 12? Of 15? Of 27? Of 30? Of 36? Of 60?

10. What is 1 *fourth* of 8? Of 16? Of 20? Of 24? Of 40? Of 48? Of 100?

11. What is 1 *fifth* of 10? Of 25? Of 30? Of 35? Of 45? Of 50? Of 55? Of 65?

12. What is 1 *sixth* of 12? Of 18? Of 30? Of 42? Of 60? Of 72? Of 90?

13. How many fourths in 1 apple?

14. How many fourths in 2 apples? In 3 apples? In 8 apples? In 16 apples?

15. How many fifths in 1 barrel of flour? In 3 barrels? In 5 barrels? In 7 barrels? In 9 barrels?

16. How many sixths in 1 bushel of wheat? In 4 bushels? In 7 bushels? In 9 bushels? In 12 bushels?

17. James owns 3 fifths of a kite, and his brother Thomas the remainder. How many fifths does Thomas own?

ILLUSTRATION. — Since there are 5 fifths in the kite, if James owns 3 fifths, there will remain for Thomas 5 fifths ($\frac{5}{5}$) less 3 fifths ($\frac{3}{5}$) = 2 fifths. Ans. 2 fifths.

18. From a load of hay I sold 4 sevenths; how many sevenths remain?

19. John Jones found a large sum of money; he gave 5 eighths of it to the poor of the parish; how much did he reserve for himself?

20. John Smith gave 2 ninths of his farm to his son, 3 ninths to his daughter, and the remainder to his wife; how many ninths did his wife receive?

ILLUSTRATION. — Since he gave 2 ninths ($\frac{2}{9}$) to his son, and 3 ninths ($\frac{3}{9}$) to his daughter, he gave them both $\frac{2}{9} + \frac{3}{9} = \frac{5}{9}$; and since there are 9 ninths ($\frac{9}{9}$) in the farm, he must have given his wife $\frac{9}{9} - \frac{5}{9} = \frac{4}{9}$. Ans. $\frac{4}{9}$.

21. In a certain school $\frac{2}{11}$ of the pupils study grammar, $\frac{3}{11}$ study arithmetic, $\frac{4}{11}$ geography, and the remainder philosophy. What part of the school study philosophy?

22. J. Dow spends $\frac{1}{4}$ of his time in reading, $\frac{3}{7}$ in labor, and $\frac{2}{7}$ in visiting. How large a portion of his time remains for eating and sleeping?

23. If a yard of cloth cost \$8, what cost $\frac{1}{4}$ of a yard? What cost $\frac{3}{4}$ of a yard?

ILLUSTRATION. — If 1 yard cost \$8, $\frac{1}{4}$ of a yard will cost $\frac{1}{4}$ of \$8 = \$2; and if $\frac{1}{4}$ of a yard cost \$2, $\frac{3}{4}$ will cost three times as much; 3 times \$2 = \$6. Ans. \$6.

24. If an acre of land cost \$24, what will $\frac{1}{8}$ of an acre cost? What will $\frac{5}{8}$ cost?

25. When 96 cents are paid for a bushel of rye, what cost $\frac{1}{12}$ of a bushel?

26. If $\frac{1}{5}$ of a barrel of flour cost 2 dollars, what cost $\frac{4}{5}$ of a barrel?

ILLUSTRATION. — If $\frac{1}{5}$ cost 2 dollars, $\frac{4}{5}$ will cost 4 times 2 dollars = \$8. Ans. \$8.

27. If $\frac{1}{8}$ of an acre of land cost \$24 dollars, what will $\frac{7}{8}$ of an acre cost?

28. If $\frac{1}{3}$ of a hogshead of molasses cost \$11, what will a hogshead cost?

29. If $\frac{7}{8}$ of an acre of land cost \$21, what cost $\frac{1}{8}$ of an acre? What cost an acre? What cost 10 acres?

ILLUSTRATION. — If $\frac{7}{8}$ cost \$21, $\frac{1}{8}$ will cost $\frac{1}{7}$ of \$21, and $\frac{1}{7}$ of \$21 is \$3; and $\frac{8}{8}$ will cost 8 times \$3 = \$24, and 10 acres will cost 10 times \$24 = \$240. Ans. \$240.

30. If $\frac{9}{11}$ of a hogshead of sugar cost \$18, what costs 1 hogshead? What cost 4 hogsheads?

31. If $\frac{5}{8}$ of a barrel of apples cost \$1.50, what costs a barrel? What cost 10 barrels?

32. When \$49 are paid for $\frac{7}{11}$ of a ton of potash, what must be paid for 2 tons?

33. How many half-barrels of flour are there in 2 and a half ($2\frac{1}{2}$) barrels?

ILLUSTRATION. — Since 1 barrel contains 2 halves, 2 barrels will contain 2 times 2 = 4 halves, and the 1 half added makes 5 halves.

34. How many half-bushels in $4\frac{1}{2}$ bushels of oats? In $5\frac{1}{2}$ bushels? In $7\frac{1}{2}$ bushels? In $9\frac{1}{2}$ bushels?

35. How many eighths of a dollar in $2\frac{1}{8}$ dollars? In $4\frac{3}{8}$ dollars? In $7\frac{5}{8}$ dollars? In $9\frac{7}{8}$ dollars? In $12\frac{1}{8}$ dollars?

36. How many tenths of an ounce in $4\frac{3}{10}$ ounces? In $5\frac{7}{10}$ ounces? In $8\frac{1}{10}$ ounces? In $10\frac{9}{10}$ ounces?

37. How many barrels of wine in 6 half ($\frac{1}{2}$) barrels?

ILLUSTRATION. — Since it takes 2 halves to make one whole one, there will be as many whole barrels in 6 halves ($\frac{6}{2}$) as 2 is contained times in 6. 2 is contained in 6, 3 times.

Ans. 3 barrels.

38. How many firkins of butter in $\frac{8}{2}$ firkins? In $\frac{20}{2}$ firkins?

39. How many whole numbers in $\frac{10}{5}$? In $\frac{15}{5}$? In $\frac{25}{5}$?

40. How many whole numbers in $\frac{16}{8}$? In $\frac{8}{8}$? In $\frac{32}{8}$? In $\frac{48}{8}$?

41. If a skein of silk is worth $3\frac{1}{2}$ cents, what are 6 skeins worth?

ILLUSTRATION. — If 1 skein is worth $3\frac{1}{2}$ cents, 6 skeins are worth 6 times as much; 6 times $3\frac{1}{2}$ are equal to 6 times 3 and 6 times $\frac{1}{2}$; 6 times $3 = 18$; 6 times $\frac{1}{2} = \frac{6}{2} = 3$; $18 + 3 = 21$.

Ans. 21 cents.

42. Bought one pair of boots for $\$6\frac{1}{2}$; what must I pay for 4 pairs? For 8 pairs? For 10 pairs? For 12 pairs?

43. Paid $12\frac{1}{2}$ cents for one pound of cloves; what will 6 pounds cost? 10 pounds? 12 pounds?

44. If one pound of butter is worth 12 cents, what are $4\frac{1}{2}$ pounds worth?

ILLUSTRATION. — If 1 pound is worth 12 cents, $4\frac{1}{2}$ pounds are worth $4\frac{1}{2}$ times as much; $4\frac{1}{2}$ times 12 cents are equal to 4 times 12 and $\frac{1}{2}$ of 12; 4 times 12 are 48, and $\frac{1}{2}$ of 12 is 6; 48 cents and 6 cents are 54 cents.

45. When lard is sold for 9 cents per pound, what must be paid for $7\frac{1}{2}$ pounds? For $8\frac{1}{2}$ pounds? For $9\frac{1}{2}$ pounds?

46. Bought 1 pound of coffee at 16 cents; what will $5\frac{1}{2}$ pounds cost? $3\frac{1}{4}$ pounds? $5\frac{1}{4}$ pounds? $6\frac{1}{8}$ pounds?

47. If 1 yard of cloth is worth 20 cents, what is the value of $16\frac{1}{2}$ yards? $12\frac{1}{4}$ yards? $8\frac{1}{2}$ yards? $11\frac{1}{4}$ yards?

48. If $1\frac{1}{2}$ bushels of corn cost \$1.20, what will 1 bushel cost?

ILLUSTRATION. — $1\frac{1}{2}$ bushels $= \frac{3}{2}$ bushels. Now, if $\frac{3}{2}$ cost \$1.20, $\frac{1}{2}$ will cost $\frac{1}{3}$ of \$1.20 = \$0.40; and $\frac{2}{2}$ or a whole bushel will cost 2 times \$0.40 = \$0.80. Ans. \$0.80.

49. If $2\frac{2}{5}$ pounds of coffee cost 60 cents, what will 1 pound *cost*?

ILLUSTRATION. — $2\frac{2}{5}$ pounds $= \frac{12}{5}$ pounds. If $\frac{12}{5}$ cost 60 cents, $\frac{1}{5}$ will cost $\frac{1}{12}$ of 60 cents $= 5$ cents; and $\frac{5}{5}$, or a pound, will cost 5 times 5 cents $= 25$ cents.

50. How many times will 60 contain $2\frac{2}{5}$?

51. Paid \$54 for $7\frac{1}{2}$ barrels of oil; what cost 1 barrel?

52. How many times is $7\frac{1}{2}$ contained in 54?

53. How many cords of wood, at $\$5\frac{1}{2}$ per cord, can be bought for \$66?

54. How many times will 66 contain $5\frac{1}{2}$?

55. Gave \$40 for $6\frac{2}{3}$ yards of broadcloth; what cost 1 yard?

56. How many times is $6\frac{2}{3}$ contained in 40?

57. The distance between two places is 110 rods. I wish to divide this distance into spaces of $5\frac{1}{2}$ rods each. Required the number of spaces.

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§ VII. CONTRACTIONS IN MULTIPLICATION AND DIVISION.*

CONTRACTIONS IN MULTIPLICATION.

ART. **61.** To multiply by 25.

EX. 1. Multiply 876581 by 25. Ans. 21914525.

OPERATION.

4) 8 7 6 5 8 1 0 0

2 1 9 1 4 5 2 5 Product.

We multiply by 100, by annexing two ciphers to the multiplicand; and since 25, the multiplier, is only *one fourth* of 100, we divide by 4 to obtain the true product.

RULE. — *Annex two ciphers to the multiplicand, and divide it by* 4.

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* If the principles on which these contractions depend are considered too difficult for the young pupil to understand at this stage of his progress, they may be omitted for the present, and attended to when he is further advanced.

QUESTIONS. — Art. 61. What is the rule for multiplying by 25? What is the reason for the rule?

EXAMPLES FOR PRACTICE.

2. Multiply 76589658 by 25. Ans. 1914741450.
3. Multiply 567898717 by 25.
4. Multiply 123456789 by 25.

ART. **62.** To multiply by $33\frac{1}{3}$.

Ex. 1. Multiply 87678963 by $33\frac{1}{3}$. Ans. 2922632100.

OPERATION.

3)87678963 00

29226321 00 Product.

We multiply by 100, as before; and since $33\frac{1}{3}$, the multiplier, is only *one third* of 100, we divide by 3 to obtain the true product.

RULE. — *Annex two ciphers to the multiplicand, and divide it by* 3.

EXAMPLES FOR PRACTICE.

2. Multiply 356789541 by $33\frac{1}{3}$. Ans. 11892984700.
3. Multiply 871132182 by $33\frac{1}{3}$.
4. Multiply 583647912 by $33\frac{1}{3}$.

ART. **63.** To multiply by 125.

Ex. 1. Multiply 7896538 by 125. Ans. 987067250.

OPERATION.

8)7896538000

987067250 Product.

We multiply by 1000, by annexing three ciphers to the multiplicand; and since 125, the multiplier, is only *one eighth* of 1000, we divide by 8 to obtain the true product.

RULE. — *Annex three ciphers to the multiplicand, and divide it by* 8

EXAMPLES FOR PRACTICE.

2. Multiply 7965325 by 125. Ans. 995665625.
3. Multiply 1234567 by 125.
4. Multiply 3049862 by 125.

QUESTIONS. — Art. 62. What is the rule for multiplying by $33\frac{1}{3}$? What is the reason for this rule? — Art. 63. What is the rule for multiplying by *125?* Give the reason for the rule.

ART. **64.** To multiply by any number of 9's.

Ex. 1. Multiply 4789653 by 99999. Ans. 478960510347.

OPERATION.

4 7 8 9 6 5 3 0 0 0 0 0
4 7 8 9 6 5 3

4 7 8 9 6 0 5 1 0 3 4 7 Product.

By adding 1 to any number composed of nines, we obtain a number expressed by 1 with as many ciphers annexed as there are nines in the number to which 1 is added. Thus, $999 + 1 = 1000$. Therefore, annexing to the multiplicand as many ciphers as there are nines in the multiplier is the same thing as multiplying the number by a multiplier too large by 1, and subtracting the number to be multiplied from this enlarged product will give the true product.

RULE. — *Annex as many ciphers to the multiplicand as there are* 9's *in the multiplier, and from this number subtract the number to be multiplied.*

EXAMPLES FOR PRACTICE.

2. Multiply 1234567 by 999. Ans. 1233332433.
3. Multiply 876543 by 999999.
4. Multiply 999999 by 999999.

CONTRACTIONS IN DIVISION.

ART. **65.** To divide by 25.

Ex. 1. Divide 1234567 by 25. Ans. $49382\frac{68}{100}$.

OPERATION.

1 2 3 4 5 6 7
4

4 9 3 8 2 | 6 8 Quotient.

Multiplying the dividend by 4 makes it four times too great; therefore, to obtain the true quotient, we must divide by 100, a divisor four times greater than the true one. This we do by cutting off two figures on the right.

RULE. — *Multiply the dividend by* 4, *and divide the product by* 100.

EXAMPLES FOR PRACTICE.

2. Divide 9876525 by 25. Ans. 395061.
3. Divide 1378925 by 25.
4. Divide 899999 by 25.

QUESTIONS. — Art. 64. What is the rule for multiplying by any number of 9's? What is the reason for the rule? — Art. 65. What is the rule for dividing by 25? Give the reason for the rule.

ART. **66.** To divide by $33\frac{1}{3}$.

EX. 1. Divide 6789543 by $33\frac{1}{3}$. Ans. $203686\frac{29}{100}$.

OPERATION.

6 7 8 9 5 4 3
3

2 0 3 6 8 6|2 9 Quotient.

Multiplying the dividend by 3 makes it three times too great; therefore, to obtain the true quotient, we must divide by 100, a divisor three times greater than the true one. This is done by cutting off two figures on the right.

RULE. — *Multiply the dividend by* 3, *and divide the product by* 100.

EXAMPLES FOR PRACTICE.

2. Divide 987654321 by $33\frac{1}{3}$. Ans. $29629629\frac{63}{100}$.
3. Divide 8712378 by $33\frac{1}{3}$.
4. Divide 4789536 by $33\frac{1}{3}$.
5. Divide 89676 by $33\frac{1}{3}$.
6. Divide 17854 by $33\frac{1}{3}$.

ART. **67.** To divide by 125.

EX. 1. Divide 9874725 by 125. Ans. $78997\frac{8}{10}$.

OPERATION.

9 8 7 4 7 2 5
8

7 8 9 9 7|8 0 0 Quotient.

Multiplying the dividend by 8 makes it eight times too great; therefore, to obtain the true quotient, we must divide by 1000, a divisor eight times greater than the true one. We do this by cutting off three figures on the right.

RULE. — *Multiply the dividend by* 8, *and divide the product by* 1000.

EXAMPLES FOR PRACTICE.

2. Divide 1728125 by 125. Ans. 13825.
3. Divide 478763250 by 125.
4. Divide 591234875 by 125.
5. Divide 489648 by 125.
6. Divide 836184 by 125.

QUESTIONS. — Art. 66. What is the rule for dividing by $33\frac{1}{3}$? Give the reason for the rule. — Art. 67. What is the rule for dividing by 125? What is the reason for the rule?

§ VIII. MISCELLANEOUS EXAMPLES,

INVOLVING THE FOREGOING RULES.

1. A BOUGHT 73 hogsheads of molasses at 29 dollars per hogshead, and sold it at 37 dollars per hogshead; what did he gain?

2. B bought 896 acres of wild land at 15 dollars per acre, and sold it at 43 dollars per acre; what did he gain?

3. N. Gage sold 47 bushels of corn at 57 cents per bushel, which cost him only 37 cents per bushel; how many cents did he gain?

4. A butcher bought a lot of beef weighing 765 pounds at 11 cents per pound, and sold it at 9 cents per pound; how many cents did he lose?

5. A taverner bought 29 loads of hay at 17 dollars per load, and 76 cords of wood at 5 dollars a cord; what was the amount of the hay and the wood?

6. Bought 17 yards of cotton at 15 cents per yard, 46 gallons of molasses at 28 cents per gallon, 16 pounds of tea at 76 cents a pound, and 107 pounds of coffee at 14 cents a pound; what was the amount of my bill?

7. A man travelled 78 days, and each day he walked 27 miles; what was the length of his journey?

8. A man sets out from Boston to travel to New York, the distance being 223 miles, and walks 27 miles a day for 6 days in succession; what distance remains to be travelled?

9. What cost a farm of 365 acres at 97 dollars per acre?

10. Bought 376 oxen at 36 dollars per ox, 169 cows at 27 dollars each, 765 sheep at 4 dollars per head, and 79 elegant horses at 275 dollars each; what was paid for all?

11. J. Barker has a fine orchard, consisting of 365 trees, and each tree produces 7 barrels of apples, and these apples will bring him in market 3 dollars per barrel; what is the income of the orchard?

12. J. Peabody bought of E. Ames 7 yards of his best broadcloth at 9 dollars per yard, and in payment he gave Ames a

one hundred-dollar bill, how many dollars must I return to him?

18. Bought of I Pierce a cooking-stove for 35 dollars, 7 cwt. of tin ware at 4 dollars per cwt., 12 pushes of rye at 1 dollar per bushel, and 3 milch cows at 31 dollars each: in part payment for the above articles I sold him eight thousand feet of boards at 11 dollars per thousand. How much must I pay him to balance the account?

19. If 1 day there are 24 hours, how many in 27 days?

20. If one pound avoirdupois weight there are 16 ounces; how many ounces are there in 245 pounds?

21. In a square mile there are 640 acres: how many acres are there in a town which contains 24 square miles?

22. What cost 75 barrels of apples at 2 dollars per barrel?

23. Bought 300 barrels of flour at 5 dollars per barrel, 47 hundred weight of cheese at 9 dollars per hundred weight, and 15 barrels of salmon at 17 dollars per barrel: what was the amount of my purchase?

24. Bought 760 acres of land at 47 dollars per acre, and sold J. Emery 171 acres at 50 dollars per acre, J. Smith 275 acres at 57 dollars per acre, and the remainder I sold at J. Kimball at 75 dollars per acre; how much did I gain by my sales?

25. Bought a hogshead of oil containing 184 gallons, at 75 cents per gallon; but 25 gallons having leaked out, I sold the remainder at 90 cents per gallon; did I gain or lose by my bargain?

26. Bought a quantity of flour, for which I gave 1728 dollars, there being 288 barrels; I sold the same at 8 dollars per barrel; how much did I gain?

27. Purchased a cargo of molasses for 9212 dollars, there being 196 hogsheads; I sold the same at 67 dollars per hogshead; how much did I gain on each hogshead?

28. A farmer bought 5 yoke of oxen at 87 dollars a yoke; 37 cows at 37 dollars each; 89 sheep at 3 dollars apiece. He sold the oxen at 98 dollars a yoke; for the cows he received 40 dollars each; and for the sheep he had 4 dollars apiece. How much did he gain by his trade?

24. The sum of two numbers is 5482, and the smaller number is 1962; what is the difference?

25. The difference between two numbers is 125, and the smaller number is 1482; what is the greater?

26. The difference between two numbers is 1282, and the greater number is 6958; what is the smaller?

27. If the dividend is 21775, and the divisor 871, what is the quotient?

28. If the quotient is 482, and the divisor 281, what is the dividend?

29. If 144 inches make 1 square foot, how many square feet in 20736 inches?

30. An acre contains 160 square rods; how many rods in a farm containing 769 acres?

31. A gentleman bought a house for three thousand forty-seven dollars, and a carriage and span of horses for five hundred seven dollars. He paid at one time two thousand seventeen dollars, and at another time nine hundred seven dollars. How much remains due?

32. The erection of a factory cost 68,255 dollars; supposing this sum to be divided into 365 shares, what is the value of each?

33. Bought two lots of wild land; the first contained 144 acres, for which I paid 12 dollars per acre; the second contained 108 acres, which cost 15 dollars per acre. I sold both lots at 18 dollars per acre; what was the amount of gain?

34. Sold 17 cords of oak wood at 6 dollars per cord, 36 cords of maple at 3 dollars per cord, and 29 cords of walnut at 7 dollars per cord. What was the amount received?

35. Daniel Bailey has a fine farm of 300 acres, which cost him 73 dollars per acre. He sold 83 acres of this farm to Minot Thayer, for 97 dollars per acre; 42 acres to J. Russel, for 87 dollars per acre; 75 acres to J. Dana, at 75 dollars per acre; and the remainder to J. Webster, at 100 dollars per acre. What was his net gain?

36. J. Gale purchased 17 sheep for 3 dollars each, 19 cows at 27 dollars each, and 47 oxen at 57 dollars each. He sold his purchase for 3700 dollars. What did he gain?

37. Purchased 17 tons of copperas at 32 dollars per ton. I sold 7 tons at 29 dollars per ton, 8 tons at 36 dollars per ton,

and the remainder at 15 dollars per ton. Did I gain or lose, and how much?

40. John Smith bought 25 yards of broadcloth at 5 dollars per yard, and 15 yards of silk at 9 dollars per yard. Did he gain or lose, and how much?

41. What is the greater value, 185 acres of land at 76 dollars per acre, or 700 hogsheads of molasses at 25 dollars per hogshead? Ans. The land is $2160 dollars.

42. Bought of J. Law 7 tons of hay at 12 dollars per ton. I paid him 75 dollars, and 12 yards of broadcloth at 4 dollars per yard. How much remains due to Law?

43. A purchased of B 34 cords of wood at 5 dollars per cord, 9 tons of hay at 17 dollars per ton. He gave him at 2 dollars apiece, 37 yards of broadcloth at 4 dollars per yard, and 16 barrels of flour at 9 dollars per barrel: what is the amount of A's bill?

44. John Smith, Jr. bought of R. S. Davis 18 dozen of National Arithmetics at 9 dollars per dozen, 23 dozen of Mental Arithmetics at 1 dollar per dozen, 17 dozen Family Bibles at 3 dollars per copy; what is the amount of the bill?

43. R. Hasseltine sold to John James 169 tons of timber at 7 dollars per ton, 116 cords of oak wood at 6 dollars per cord, and 37 cords of maple wood at 5 dollars per cord; James has paid Hasseltine 144 dollars in cash, and 23 yards of cloth at 4 dollars per yard; what remains due to Hasseltine?

44. J. Frost owes me on account 375 dollars, and he has paid me 6 cords of wood at 5 dollars per cord, 15 tons of hay at 12 dollars per ton, and 32 bushels of rye at 1 dollar per bushel. How much remains due to me?

45. Gave 169 dollars for a chaise, 87 dollars for a harness, and 176 dollars for a horse. I sold the chaise for 187 dollars, the harness for 107 dollars, and the horse for 165 dollars. What sum have I gained?

46. Bought a farm of J. C. Bradbury for 1728 dollars, for which I paid him 75 barrels of flour at 6 dollars per barrel, 9 cords of wood at 5 dollars a cord, 17 tons of hay at 25 dollars a ton, 40 bushels of wheat at 2 dollars a bushel, and 65 bushels of beans at 3 dollars a bushel; how many dollars remain due to Bradbury?

§ IX. UNITED STATES MONEY.

ART. 68. UNITED STATES MONEY, established by Congress in 1796, is the legal currency of the United States.

TABLE.

10 Mills	make	1 Cent,	marked	c.
10 Cents	"	1 Dime,	"	d.
10 Dimes	"	1 Dollar,	"	$.
10 Dollars	"	1 Eagle,	"	E.

				Dimes.		Cents.		Mills.
				1	=	10	=	10
		Dollars.		1	=	10	=	100
	Eagle.	1	=	10	=	100	=	1000
	1	10	=	100	=	1000	=	10000

SIMPLE NUMBERS, that is, numbers whose units are all of a single denomination, have thus far, in this work, been made use of alone in the operations.

But as the units or denominations of United States money increase from right to left, and decrease from left to right, in the same manner as do the units of the several orders in simple numbers, they may, therefore, be added, subtracted, multiplied, and divided, according to the same rules.

Dollars are separated from cents by a point, called a separatrix or decimal point; the first two places at the right of the point being cents; and the third place, mills. Thus, $16.253 is read, sixteen dollars, twenty-five cents, three mills.

Since cents occupy two places, the place of dimes and of cents, when the number of cents is less than 10, a cipher must be written before them in the place of dimes; thus, .03, .07, &c.

The *coins* of the United States consist of the double-eagle, eagle, half-eagle, quarter-eagle, three dollars, and dollar, made of *gold*; the dollar, half-dollar, quarter-dollar, dime, half-dime, and three-cent piece, made of *silver*; the cent and half-cent, made of *copper*.

NOTE 1. — The word MILL is from the Latin word *mille* (one thousand); the word CENT, from the Latin *centum* (one hundred); the word DIME, from a French word signifying a *tithe* or *tenth*; and the reason of these

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QUESTIONS. — Art. 68. What is United States money? Repeat the Table of United States Money. What is a simple number? What are the denominations of United States money? How do they increase from right to left? How are they added, subtracted, multiplied, and divided? How are dollars, cents, and mills, separated? Why must a cipher be placed before cents, when the number is less than 10? Why are two places allowed for cents, while only one is allowed for mills? Name the coins of the United States.

names, as applied to our coins, is found in the proportion which they respectively bear to the dollar.

The term DOLLAR is said to be derived from the Danish word *Daler*, and this from *Dale*, the name of a town, where it was first coined.

The symbol $ represents, probably, the letter U written upon an S, denoting U. S. (United States).

NOTE 2. — All the gold and silver coins of the United States are now made of one purity, nine parts of pure metal, and one part alloy. The alloy for the silver is pure copper; and that for the gold, one part copper and one part silver. The cent is now made of pure copper and nickel. The standard weight, as fixed by present laws, of the eagle, is 258 grains, Troy; the silver dollar, 412½ grains; half-dollar, 192 grains; quarter-dollar, 96 grains; dime, 38$\frac{2}{5}$ grains; half-dime, 19$\frac{1}{5}$ grains; three-cent piece, 11$\frac{52}{100}$ grains; and the cent, new coinage, 72 grains.

REDUCTION OF UNITED STATES MONEY.

ART. **69.** REDUCTION of United States Money is changing the units of one of its denominations to the units of another, either of a higher or lower denomination, without altering their value.

ART. **70.** To reduce units from a higher denomination to a lower.

Ex. 1. Reduce 25 dollars to cents and mills.

Ans. 2500 cents, 25000 mills

OPERATION.

25 dollars.
100
2500 cents.
10
25000 mills.

We multiply the 25 by 100, because 100 cents make 1 dollar; and multiply the 2500 by 10, because 10 mills make 1 cent.

Or thus, 25000 mills.

RULE. — *To reduce dollars to cents, annex* TWO *ciphers; to reduce dollars to mills, annex* THREE *ciphers; and to reduce cents to mills, annex* ONE *cipher.*

NOTE. — Dollars, cents, and mills, expressed by a single number, are reduced to mills by merely removing the separating point; and dollars and cents, by annexing one cipher and removing the separatrix.

ART. **71.** To reduce units from a lower denomination to a higher.

Ex. 1. Reduce 25000 mills to cents and dollars.

Ans. 2500 cents, $25.

QUESTIONS. — Art. 69. What is reduction of United States Money? — Art. 70. What is the rule for reducing dollars to cents and mills? Give the reason for the rule. How do you reduce dollars and cents to cents, or dollars, cents *and mills, to mills*? What is the reason for this rule?

OPERATION.

10) 25000 mills.
100) 2500 cents.
25 dollars.

We divide the 25000 by 10, because 10 mills make 1 cent; and divide the 2500 by 100, because 100 cents make 1 dollar.

Or thus, 25|0 0|0 mills.

RULE. — *To reduce mills to cents, cut off* ONE *figure on the right; to reduce cents to dollars, point off* TWO *figures; and to reduce mills to dollars, point off* THREE *figures.*

EXAMPLES FOR PRACTICE.

1. Reduce $125 to cents. Ans. 12500 cents.
2. Reduce $345 to mills. Ans. 345000 mills.
3. Reduce 297 mills to cents.
4. Reduce 2682 mills to dollars.
5. Reduce 4123 cents to dollars.
6. Reduce $156.29 to cents.
7. Reduce $16.428 to mills.
8. Reduce $9.87 to mills.

ART. 72. ADDITION OF UNITED STATES MONEY.

RULE. — *Write dollars, cents, and mills, so that units of the same denomination shall stand in the same column.*

Add as in addition of simple numbers, and place the separating point directly under that above.

Proof. — The proof is the same as in addition of simple numbers.

EXAMPLES FOR PRACTICE.

	1.	2.	3.	4.
	$. cts. m.	$. cts. m.	$. cts. m.	$. cts.
	45.243	75.643	16.705	147.86
	13.896	16.897	14.003	789.58
	93.516	43.816	18.719	496.37
	52.343	58.313	97.009	911.34
Ans.	204.998	194.669	146.436	2345.15

QUESTIONS. — Art. 71. What is the rule for reducing mills to cents? For reducing cents to dollars? For reducing mills to dollars? Give the reason for each. — Art. 72. How must the numbers be written down in addition of United States money? How added? How pointed off? Repeat the rule.

5.	6.	7.	8.
$. cts. m.	$. cts. m.	$. cts. m.	$. cts. m.
7 8 6.7 1 3	8 7.0 5 9	9 1.7 6 3	7 8 6.7 1 3
1 7 6.0 7 1	3 7.8 1 0	8 4.1 6 1	3 4 5.6 7 8
5 6 7.8 1 9	8 1.4 7 5	1 0.0 7 0	9 0 7.0 1 7
1 2 3.4 5 6	4 0.0 7 8	5 3.6 1 5	8 6 1.0 9 0
7 8 9.0 1 2	2 1.1 5 6	8 1.1 7 6	1 2 3.4 7 6
3 4 5.6 7 8	8 1.1 7 7	3 2.8 1 7	9 8 7.0 1 6
9 0 1.2 3 4	3 3.6 2 1	5 3.1 9 6	3 4 5.7 0 5
7 1 8.9 0 5	2 8.0 9 3	4 1.5 7 0	3 5 7.0 9 1

9. Bought a coat for $17.81, a vest for $3.75, a pair of pantaloons for $2.87, and a pair of boots for $7.18; what was the amount?

10. Sold a load of wood for seven dollars six cents, five bushels of corn for four dollars seventy-five cents, and seven bushels of potatoes for two dollars six cents; what was received for the whole?

11. Bought a barrel of flour for $6.50, a box of sugar for $9.87, a ton of coal for $12.77, and a box of raisins for $2.50; what was paid for the various articles?

12. Paid $4.62 for a hat, $9.75 for a coat, $5.75 for a pair of boots, and $1.50 for an umbrella; what was paid for the whole?

13. A grocer sold a pound of tea for $0.625; 4 pounds of butter for $0.75; 4 dozen of lemons for $0.875; 9 pounds of sugar for $0.80; and 3 pounds of dates for $0.375. What was the amount of the bill?

14. A student purchased a Latin grammar for $0.75, a Virgil for $3.75, a Greek lexicon for $4.75, a Homer for $1.25, an English dictionary for $3.75, and a Greek Testament for $0.75; what was the amount of the bill?

15. Bought of J. H. Carleton a China tea-set for ten dollars eighty-two cents, a dining-set for nine dollars sixty-two cents five mills, a solar lamp for ten dollars fifty cents, a pair of vases for four dollars sixty-two cents five mills, and a set of silver spoons for twelve dollars seventy-five cents; what did the whole cost?

16. Bought three hundred weight of beef at seven dollars seven cents per hundred weight, four cords of wood at six dollars four cents per cord, and a cheese for three dollars nine cents; what was the amount of the bill?

ART. 73. SUBTRACTION OF UNITED STATES MONEY.

RULE. — *Write the several denominations of the subtrahend under the corresponding ones of the minuend.*

Subtract as in subtraction of simple numbers, and place the separatrix directly under that above.

Proof. — The proof is the same as in subtraction of simple numbers.

EXAMPLES FOR PRACTICE.

	1.	2.	3.	4.
	$. cts. m.	$. cts.	$. cts. m.	$. cts.
Min.	61.585	471.81	156.003	141.70
Sub.	19.197	158.19	19.009	90.91
Rem.	42.388	313.62	136.994	50.79

	5.	6.	7.	8.
	$. cts. m.	$. cts. m.	$. cts. m.	$. cts. m.
From	71.861	91.071	815.701	10781.303
Take	19.197	19.095	90.803	9999.097

9. From $71.07 take $5.09.

10. From $100 take $17.17.

11. From one hundred dollars there were paid to one man seventeen dollars nine cents, to another twenty-three dollars eight cents, and to another thirty-three dollars twenty-five cents; how much cash remained?

12. From ten dollars take nine mills.

13. A lady went "a shopping," her mother having given her fifty dollars. She purchased a dress for fifteen dollars seven cents; a shawl for eleven dollars ten cents; a bonnet for seven dollars nine cents; and a pair of shoes for two dollars. How much money had she remaining?

14. From one hundred dollars there were taken at one time thirty-one dollars fifteen cents seven mills; at another time, seven dollars nine cents five mills; at another time, five dollars five cents; and at another time, twenty-two dollars two cents seven mills. How much cash remained of the hundred dollars?

QUESTIONS. — Art. 73. How do you write down the numbers in subtraction of United States money? How subtract? How pointed off? Repeat the rule.

ART. **66.** To divide by $33\frac{1}{3}$.

EX. 1. Divide 6789543 by $33\frac{1}{3}$. ANS. $203686\frac{29}{100}$.

OPERATION.

6 7 8 9 5 4 3
3

2 0 3 6 8 6|2 9 Quotient.

Multiplying the dividend by 3 makes it three times too great; therefore, to obtain the true quotient, we must divide by 100, a divisor three times greater than the true one. This is done by cutting off two figures on the right.

RULE. — *Multiply the dividend by* 3, *and divide the product by* 100.

EXAMPLES FOR PRACTICE.

2. Divide 987654321 by $33\frac{1}{3}$. ANS. $29629629\frac{63}{100}$.
3. Divide 8712378 by $33\frac{1}{3}$.
4. Divide 4789536 by $33\frac{1}{3}$.
5. Divide 89676 by $33\frac{1}{3}$.
6. Divide 17854 by $33\frac{1}{3}$.

ART. **67.** To divide by 125.

EX. 1. Divide 9874725 by 125. ANS. $78997\frac{8}{10}$.

OPERATION.

9 8 7 4 7 2 5
8

7 8 9 9 7|8 0 0 Quotient.

Multiplying the dividend by 8 makes it eight times too great; therefore, to obtain the true quotient, we must divide by 1000, a divisor eight times greater than the true one. We do this by cutting off three figures on the right.

RULE. — *Multiply the dividend by* 8, *and divide the product by* 1000.

EXAMPLES FOR PRACTICE.

2. Divide 1728125 by 125. ANS. 13825.
3. Divide 478763250 by 125.
4. Divide 591234875 by 125.
5. Divide 489648 by 125.
6. Divide 836184 by 125.

QUESTIONS. — Art. 66. What is the rule for dividing by $33\frac{1}{3}$? Give the reason for the rule. — Art. 67. What is the rule for dividing by 125? What is the reason for the rule?

§ VIII. MISCELLANEOUS EXAMPLES,

INVOLVING THE FOREGOING RULES.

1. A BOUGHT 73 hogsheads of molasses at 29 dollars per hogshead, and sold it at 37 dollars per hogshead; what did he gain?

2. B bought 896 acres of wild land at 15 dollars per acre, and sold it at 43 dollars per acre; what did he gain?

3. N. Gage sold 47 bushels of corn at 57 cents per bushel, which cost him only 37 cents per bushel; how many cents did he gain?

4. A butcher bought a lot of beef weighing 765 pounds at 11 cents per pound, and sold it at 9 cents per pound; how many cents did he lose?

5. A taverner bought 29 loads of hay at 17 dollars per load, and 76 cords of wood at 5 dollars a cord; what was the amount of the hay and the wood?

6. Bought 17 yards of cotton at 15 cents per yard, 46 gallons of molasses at 28 cents per gallon, 16 pounds of tea at 76 cents a pound, and 107 pounds of coffee at 14 cents a pound; what was the amount of my bill?

7. A man travelled 78 days, and each day he walked 27 miles; what was the length of his journey?

8. A man sets out from Boston to travel to New York, the distance being 223 miles, and walks 27 miles a day for 6 days in succession; what distance remains to be travelled?

9. What cost a farm of 365 acres at 97 dollars per acre?

10. Bought 376 oxen at 36 dollars per ox, 169 cows at 27 dollars each, 765 sheep at 4 dollars per head, and 79 elegant horses at 275 dollars each; what was paid for all?

11. J. Barker has a fine orchard, consisting of 365 trees, and each tree produces 7 barrels of apples, and these apples will bring him in market 3 dollars per barrel; what is the income of the orchard?

12. J. Peabody bought of E. Ames 7 yards of his best broadcloth at 9 dollars per yard, and in payment he gave Ames a

5. Gave $22.50 for 18 barrels of apples; what was paid for 1 barrel? For 5 barrels? For 10 barrels?

6. Bought 153 pounds of tea for $90.27; what was it per pound?

7. A merchant purchased a bale of cloth, containing 73 yards, for $414.64; what was the cost of 1 yard?

8. If 126 pounds of butter cost $16.38, what will 1 pound cost?

9. If 63 pounds of tea cost $58.59, what will 1 pound cost?

10. If 76 cwt. of beef cost $249.28, what will 1 cwt. cost?

11. If 96,000 feet of boards cost $1120.32, what will a thousand feet cost?

12. Sold 169 tons of timber for $790.92; what was received for 1 ton?

13. When 369 tons of potash are sold for $48910.95, what is received for 1 ton?

14. For 19 cords of wood I paid $109.25; what was paid for 1 cord?

PRACTICAL QUESTIONS BY ANALYSIS.

ART. **76.** ANALYSIS is an examination of a question by resolving it into its parts, in order to consider them separately, and thus render each step in the solution plain and intelligible.

ART. **77.** The price of one pound, yard, bushel, &c., being given, to find the price of any quantity.

RULE. — *Multiply the price by the quantity.*

Ex. 1. If 1 ton of hay cost $12, what will 29 tons cost?
Ans. $348.

ILLUSTRATION. — Since 1 ton costs $12, 29 tons will cost 29 times as much: $\$12 \times 29 = \$348$.

2. If 1 bushel of salt cost 93 cents, what will 40 bushels cost? What will 97 bushels cost? Ans. $90.21.

QUESTIONS. — Art. 77. The price of 1 pound, &c., being given, how do you find the price of any quantity? Give the reason for this rule.

3. If 1 bushel of apples cost $1.65, what will 5 bushels cost? What will 18 bushels cost?

4. If 1 ton of clay cost $0.67, what will 7 tons cost? What will 63 tons cost?

5. When $7.83 are paid for 1 cwt. of sugar, what will 12 cwt. cost? What will 93 cwt. cost?

6. When $0.09 are paid for 1 lb. of beef, what will 12 lb. cost? What will 760 lb. cost?

7. A gentleman paid $38.37 for 1 acre of land; what was the cost of 20 acres. What would 144 acres cost?

8. Paid $6.83 for 1 barrel of flour; what was the value of 9 barrels? What must be paid for 108 barrels?

ART. 78. The price of any quantity, and the quantity being given, to find the price of a unit of that quantity.

RULE. — *Divide the price by the quantity.*

9. If 15 bushels of corn cost $10.35, what will 1 bushel cost? Ans. $0.69.

ILLUSTRATION. — Since 15 bushels cost $10.35, 1 bushel will cost as many cents as 15 is contained times in $10.35 : $10.35 ÷ 15 = $0.69.

10. Bought 65 barrels of flour for $422.50; what cost one barrel? What cost 15 barrels? Ans. $97.50.

11. For 45 acres of land a farmer paid $2025; what cost 1 acre? What 180 acres?

12. For 5 pairs of gloves a lady paid $3.45; what cost 1 pair? What cost 11 pairs?

13. If 11 tons of hay cost $214.50, what will 1 ton cost? What will 87 tons cost?

14. When $60 are paid for 8 dozen of arithmetics, what will 1 dozen cost? What will 87 dozen cost?

15. Gave $5.58 for 9 bushels of potatoes; what will 1 bushel cost? What will 43 bushels cost?

16. Bought 5 tons of hay for $85; what would 1 ton cost? What would 97 tons cost?

QUESTIONS. — Art. 78. How do you find the price of 1 pound, &c., the price of any quantity and the quantity being given? What is the reason for this rule?

17. If J. Ladd will sell 20 lb. of butter for $3.80, what should he charge for 59 lb.?

18. Sold 27 acres of land for $472.50; what was the price of 1 acre? What should be given for 12 acres?

19. Paid $39.69 for 7 cords of wood; what will 1 cord cost? What will 57 cords cost?

20. Paid $10.08 for 144 lb. of pepper; what was the price of 1 pound? What cost 359 lb.?

21. Paid $77.13 for 857 lb. of rice; what cost 1 lb.? What cost 359 lb.?

22. J. Johnson paid $187.53 for 987 gal. of molasses; what cost 1 gal.? What cost 329 gal.?

23. For 47 bushels of salt J. Ingersoll paid $26.32; what cost 1 bushel? What cost 39 bushels?

ART. **79.** The price of any quantity and the price of a unit of that quantity being given, to find the quantity.

RULE. — *Divide the whole price by the price of a unit of the quantity required.*

24. If I expend $150 for coal at $6 per ton, how many tons can I purchase? Ans. 25 tons.

ILLUSTRATION. — Since I pay $6 for 1 ton, I can purchase as many tons with $150 as $6 is contained times in $150 : $150 ÷ $6 = 25; therefore I can purchase 25 tons.

25. At $5 per ream, how many reams of paper can be bought for $175? Ans. 35 reams.

26. At $7.50 per barrel, how many barrels of flour can be obtained for $217.50?

27. At $75 per ton, how many tons of iron can be purchased for $4875?

28. At $4 per yard, how many yards of cloth can be bought for $1728?

29. How many hundred weight of hay can be bought for $9.66, if $0.69 are paid for 1 hundred weight?

30. If $66.51 are paid for flour at $7.39 per barrel, how many barrels can be bought?

31. Paid $136.50 for wood, at $3.25 per cord; how many cords did I buy?

QUESTIONS. — Art. 79. How do you find the quantity, the price of 1 pound, &c., being given? Give the reason for the rule.

BILLS.

ART. 80. A BILL is a paper, given by merchants, containing a statement of goods sold, and their prices.

An *invoice* is a bill of merchandise shipped or forwarded to a purchaser, or selling agent.

The *date* of a bill is the time and place of the transaction.

The bill is *against* the party owing, and in *favor* of the party who is to receive the amount due.

A bill is receipted, when the receiving of the amount due is acknowledged by the party in whose favor it is. A clerk, or any other authorized person, may, in his stead, receipt for him, as in bill 2.

When the items of a bill have been rendered at different dates, the several times may be given at the left hand, as in bill 5.

When the bill is in the form of an account, containing items of debt and credit in its settlement, it is required to find the difference due, or balance, as in bill 5.

What is the cost of each article in, and the amount due of, each of the following bills?

(1.) *New York, May* 20, 1856.

Dr. JOHN SMITH,

Bought of SOMES & GRIDLEY,

82 *gals. Temperance Wine,*	*at*	$0.75	
89 " *Port* *do.*	"	.92	
24 *pairs Silk Gloves,*	"	.50	
			$155.38.

Received payment,

SOMES & GRIDLEY.

(2.) *Philadelphia, March* 7, 1857.

Mr. LEVI WEBSTER,

Bought of JAMES FRANKLAND,

6 *lbs. Chocolate,*	*at*	$0.18
12 " *Flour,*	"	.20
6 *pairs Shoes,*	"	1.80
30 *lbs. Candles,*	"	.26
		$22.08.

Received payment,

JAMES FRANKLAND,
by ENOCH OSGOOD.

QUESTIONS. — Art. 80. What is a bill, in mercantile transactions? What is an invoice? When is a bill against, and when in favor of a party? How is a bill receipted?

(3.) *St. Louis, March* 19, 1856.

Mr. WILLIAM GREENLEAF,

Bought of MOSES ATWOOD,

86	*Shovels,*	*at*	$0.50	
90	*Spades,*	"	.86	
18	*Ploughs,*	"	11.00	
23	*Handsaws,*	"	3.50	
14	*Hammers,*	"	.62	
12	*Mill-saws,*	"	12.12	
46	*cwt. Iron,*	"	12.00	
				$1105.02.

(4.) *Boston, June* 5, 1856.

Mr. AMOS DOW,

Bought of LORD & GREENLEAF,

37	*Chests Green Tea,*	*at*	$23.75	
42	" *Black do.,*	"	17.50	
43	*Casks Wine,*	"	99.00	
12	*Crates Liverpool Ware,*	"	175.00	
19	*bbl. Genesee Flour,*	"	7.00	
23	*bu. Rye,*	"	1.52	
				$8138.71.

(5.) *San Francisco, May* 13, 1856.

Mr. JOHN WADE,

1855. *To* AYER, FITTS & Co., *Dr.*

Apr. 5.	*To* 80	*pairs Hose,*	*at*	$1.20	
Aug. 7.	" 17	" *Boots,*	"	3.00	
	" 19	" *Shoes,*	"	1.08	
Nov. 1.	" 23	" *Gloves,*	"	.75	
					$184.77.

1856. *Cr.*

Jan. 1.	27	*Young Readers,*	*at*	$0.20	
" "	10	*Greek Lexicons,*	"	3.90	
Feb. 10.	7	*Webster's Dictionaries,*	"	4.75	
Apr. 3.	19	*Folio Bibles,*	"	2.93	
" "	20	*Testaments,*	"	.37	
					$140.72.

Balance due A., F. & Co. $44.05.

Received payment,

AYER, FITTS & Co.

LEDGER ACCOUNTS.

ART. 81. The principal book of accounts among merchants is called a ledger. In it are brought together scattered items of account, often making long columns. As a rapid way of finding the amount of each, accountants generally add more than one column at a single operation. (Art. 24.) The examples below may be added both by the usual method and by that which is more rapid.

1.	2.	3.	4.
$. cts.	$. cts.	$. cts.	$. cts.
5.75	1.05	71.10	100.88
3.15	7.08	35.60	320.12
6.37	6.38	21.40	280.47
10.13	5.50	100.50	151.53
5.05	3.25	62.75	.92
12.50	8.19	13.13	11.08
8.00	1.13	1.37	49.13
.63	10.10	16.02	44.22
1.37	15.25	19.28	60.81
22.00	13.45	163.35	52.75
16.05	6.17	620.50	35.15
1.19	.09	75.00	70.06
.31	1.13	25.20	1050.00
10.00	8.07	53.81	3120.12
11.88	11.06	33.19	200.50
.12	35.15	17.00	16.09
9.17	18.91	10.38	900.11
.33	10.03	40.12	1825.50
6.22	30.00	15.68	105.10
2.31	1.88	71.12	35.46
7.17	2.75	13.19	67.63
15.50	1.25	10.00	81.17
11.25	5.00	18.20	10.14
.09	25.50	13.15	75.00
21.17	12.02	25.00	120.00
32.00	19.17	102.55	114.09
14.06	32.43	111.10	212.63
20.50	46.37	235.83	1030.48

QUESTIONS. — Art. 81. What is a ledger? How may ledger columns be added rapidly?

§ X. REDUCTION.

ART. 82. A SIMPLE number is a unit or a collection of units, either abstract, or concrete of a single kind or denomination: thus, 1 dollar, 9 apples, 12, are simple numbers.

A COMPOUND number is a collection of concrete units of several kinds or denominations, taken collectively; thus, 12£. 18s. 9d., is a compound number.

ART. 83. Reduction is changing numbers, either simple or compound, from one denomination to another, without altering their values.

It is of two kinds, Reduction Descending, and Reduction Ascending.

Reduction Descending is changing numbers of a higher denomination to a lower denomination; as pounds to shillings, &c. It is performed by multiplication.

Reduction Ascending is changing numbers of a lower denomination to a higher denomination; as farthings to pence, &c. It is the reverse of Reduction Descending, and is performed by division.

ENGLISH MONEY.

ART. 84. English or Sterling Money is the Currency of England.

TABLE.

4 Farthings (qr. or far.)	make	1 Penny,	d.
12 Pence	"	1 Shilling,	s.
20 Shillings	"	1 Pound,	£.
21 Shillings sterling	"	1 Guinea,	G.
20 Shillings "	"	1 Sovereign,	sov.

			s.		d.		far.
			1	=	1	=	4
£.			1	=	12	=	48
1	=		20	=	240	=	960

NOTE 1. — The symbol £. stands for the Latin word *libra*, signifying a pound; *s.* for *solidus*, a shilling; *d.* for *denarius*, a penny; *qr.* for *quadrans*, a quarter.

QUESTIONS. — Art. 82. What is a simple number? What is a compound number? — Art. 83. What is reduction? How many kinds of reduction? What are they? What is reduction descending? What is reduction ascending? — Art. 84. What is English money? Repeat the table.

NOTE 2. — Farthings are sometimes expressed in a fraction of a penny; thus, 1 *far.* = ¼ d.; 2 far. = ½ d.; 3 far. = ¾ d.

NOTE 3. — The *Pound Sterling* is represented by a gold coin called a sovereign. Its legal value in United States money is $4.84.

NOTE 4. — The term sterling is probably from Easterling, the popular name of certain early German traders in England, whose money was noted for the purity of its quality.

MENTAL EXERCISES.

1. How many farthings in 3 pence? In 9 pence?
2. How many pence in 2 shillings? In 6 shillings?
3. How many shillings in 7 pounds? In 10 pounds?
4. How many pence in 8 farthings? In 24 farthings?
5. How many shillings in 24 pence? In 60 pence?
6. How many pounds in 40 shillings? In 80 shillings?

EXERCISES FOR THE SLATE.

ART. 85. To reduce units of a higher denomination to a lower.

1. How many farthings in 17£. 8s. 9d. 3far.?

OPERATION.

17£. 8s. 9d. 3far.
20
348 shillings.
12
4185 pence.
4
Ans. 16743 farthings.

We multiply the 17 by 20, because 20 shillings make 1 pound, and to this product we add the 8 shillings in the question. We then multiply by 12, because 12 pence make 1 shilling, and to the product we add the 9d. Again, we multiply by 4, because 4 farthings make 1 penny, and to this product we add the 3 far.; and we find the answer to be 16743 farthings.

RULE. — *Multiply the highest denomination given by the number required of the next lower denomination to make one in the denomination multiplied. To this product add the corresponding denomination of the multiplicand, if there be any. Proceed in this way, till the reduction is brought to the denomination required.*

QUESTIONS. — Art. 85. How do you reduce pounds to shillings? Why multiply by 20? How do you reduce shillings to pence? Why? Pence to farthings? Why? Guineas to shillings? What is the general rule for reduction descending?

ART. 86. To reduce the unit of a lower denomination to a higher.

Ex. 2. How many pounds in 16743 farthings?

OPERATION.

4) 16743 far.
12) 4185 d. 3far.
20) 348 s. 9d.
17 £. 8s.

Ans. 17£. 8s. 9d. 3far.

We divide by 4, because 4 farthings make 1 penny, and the result is 4185 pence, and the remainder, 3, is farthings. We divide by 12, because 12 pence make 1 shilling, and the result is 348 shillings, and the 9 remaining is pence. Lastly, we divide by 20, because 20 shillings make 1 pound, and the result is 17£. 8s. Therefore, by annexing all the remainders to the last quotient, we find the answer to be 17£. 8s. 9d. 3far.

RULE. — *Divide the lower denomination given by the number, which it takes of that denomination to make one of the next higher. The quotient thus obtained divide as before, and so proceed until it is brought to the denomination required. The last quotient, with the remainders connected, will be the answer.*

3. In 9£. 18s. 7d. how many pence?
4. In 2383d. how many pounds, &c.?
5. How many farthings in 14£. 11s. 5d. 2far.?
6. How many pounds in 13990far.?

TROY WEIGHT.

ART. 87. Troy Weight is the weight used in weighing gold silver, and jewels.

TABLE.

24 Grains (gr.)			make	1 Pennyweight,			pwt.
20 Pennyweights			"	1 Ounce,			oz.
12 Ounces			"	1 Pound,			lb.
		oz.		pwt. 1	=	gr. 24	
lb.		1	=	20	=	480	
1	=	12	=	240	=	5760	

QUESTIONS. — Art. 86. How do you reduce farthings to pence? Why divide by 4? How do you reduce pence to shillings? Why? Shillings to pounds? Why? Shillings to guineas? What is the general rule for reduction ascending? What is Troy Weight used for? Repeat the Table.

NOTE 1. — The *oz.* stands for *onza*, the Spanish for ounce.

NOTE 2. — A grain or corn of wheat, gathered out of the middle of the ear, was the origin of all the weights used in England. Of these grains, 32, well dried, were to make one pennyweight. But in later times it was thought sufficient to divide the same pennyweight into 24 equal parts, still called grains, being the least weight now in use, from which the rest are computed.

NOTE 3. — Diamonds and other precious stones are weighed by what is called *Diamond Weight*, of which 16 *parts* make 1 *grain*; 4 grains, 1 *carat*. 1 grain Diamond Weight is equal to $\frac{4}{5}$ grains Troy, and 1 carat to $3\frac{1}{5}$ grains Troy.

NOTE 4. — The Troy pound is the *standard unit of weight* adopted by the United States Mint, and is the same as the Imperial Troy pound of Great Britain.

MENTAL EXERCISES.

1. How many gr. in 2pwt.? In 10pwt.?
2. How many pennyweights in 4oz.? In 20oz.?
3. How many ounces in 2lb.? In 5lb.? In 10lb.?
4. How many pennyweights in 48gr.? In 96gr.?
5. How many ounces in 40pwt.? In 120pwt.?
6. How many pounds in 24oz.? In 60oz.? In 120oz.?

EXERCISES FOR THE SLATE.

1. How many grains in 72lb. 10oz. 15pwt. 7gr.?

OPERATION.

72 lb. 10oz. 15pwt. 7gr.
12
874 ounces.
20
17495 pennyweights.
24
69987
34990
Ans. 419887 grains.

2. In 419887 grains, how many pounds?

OPERATION.

24)419887 gr.
20)17495 pwt. 7gr.
12)874 oz. 15pwt.
72 lb. 10oz.
Ans. 72lb. 10oz. 15pwt. 7gr.

QUESTIONS. — What was the original of all weights in England? How many of these grains did it take to make a pennyweight? How many grains in a pennyweight now? By what weight are diamonds weighed? What is the standard at the mint? How do you reduce pounds to grains? Give the reason of the operation. How do you reduce grains to pounds?

3. How many grains in 76pwt. 12gr.?
4. How many pennyweights in 1836gr.?
5. In 76lb. 5oz. how many grains?
6. In 440160 grains how many pounds?
7. How many pennyweights in 144lb. 9oz.?
8. How many pounds in 34740pwt.?
9. How many pounds in 17895gr.?
10. In 3lb. 1oz. 5pwt. 15gr. how many grains?
11. A valuable gem weighing 2oz. 18pwt. 12gr. was sold for $1.37 per grain; what was the sum paid?

APOTHECARIES' WEIGHT.

ART. 88. Apothecaries' Weight is used in mixing medicines.

TABLE.

20 Grains (gr.)	make	1 Scruple,	sc. or ℈
3 Scruples	"	1 Dram,	dr. or ʒ
8 Drams	"	1 Ounce,	oz. or ℥
12 Ounces	"	1 Pound,	lb. or ℔

					dr.		sc. 1	=	gr. 20
		oz.		1	=	3	=	60
lb.		1	=	8	=	24	=	480
1	=	12	=	96	=	288	=	5760

NOTE 1. — In this weight the pound, ounce, and grain are the same as in Troy Weight.

NOTE 2. — Medicines are usually bought and sold by Avoirdupois Weight.

NOTE 3. — In estimating the weight of fluids, 45 drops, or a common tea-spoonful, make about 1 fluid dram; 2 common table-spoonfuls, about 1 fluid ounce.

MENTAL EXERCISES.

1. In 40 grains how many scruples? In 60gr.? In 120gr.?
2. In 5 scruples how many grains? In 10sc.? In 40sc.?
3. In 3 drams how many scruples? In 10dr.? In 17dr.?
4. How many pounds in 48 ounces? In 96oz.? In 144oz.?
5. How many ounces in 24 drams? In 64dr.? In 96dr.?

QUESTIONS. — Art 88. For what is Apothecaries' Weight used? What denominations of this weight are the same as those of Troy Weight? By what weight are medicines usually bought and sold? Repeat the table.

EXERCISES FOR THE SLATE.

1. In 40lb. 8oz. 5dr. 1sc. 7gr. how many grains?

OPERATION.

40 lb. 8oz. 5dr. 1sc. 7gr.
12
488 ounces.
8
3909 drams.
3
11728 scruples.
20
Ans. 234567 grains.

2. How many pounds in 234567 grains?

OPERATION.

20)234567 gr.
3)11728 sc. 7gr
8)3909 dr. 1sc
12)488 oz. 5dr.
40 lb. 8oz.
Ans. 40lb. 8oz. 5dr. 1sc. 7gr.

3. How many scruples in 76lb.?
4. How many pounds in 21888ʒ?
5. How many grains in 144lb.?
6. How many pounds in 829440gr.?
7. In 12℔ 8℥ 3℥ 1ʒ 18gr. how many grains?
8. In 73178 grains how many pounds?
9. How many doses are there in 7℥ 6℥ 2ʒ of tartar emetic, admitting 20 grains for each dose?

AVOIRDUPOIS WEIGHT.

ART. 89. Avoirdupois Weight is used in weighing almost every kind of goods, and all metals except gold and silver.

TABLE.

16 Drams (dr.)						make	1 Ounce,						oz.
16 Ounces						"	1 Pound,						lb.
25 Pounds						"	1 Quarter,						qr.
4 Quarters						"	1 Hundred Weight,						cwt.
20 Hundred Weight						"	1 Ton,						T.
									oz.				dr.
							lb.		1	=			16
				qr.			1	=	16	=			256
		cwt.		1	=		25	=	400	=			6400
T.		1	=	4	=		100	=	1600	=			25600
1	=	20	=	80	=		20000	=	32000	=			51200

QUESTIONS. — How do you reduce pounds to grains? What is the reason for the operation? How do you reduce grains to pounds? Give the reason of the operation. — Art 89. For what is Avoirdupois Weight used? Recite the table.

NOTE 1. — In *cwt.* the *c* stands for *centum*, the Latin for *one hundred*, and *wt* for *weight*.

NOTE 2. — The laws of most of the States, and common practice at the present time, make 25 pounds a quarter, as given in the table. But formerly, 28 pounds were allowed to make a quarter, 112 pounds a hundred, and 2240 pounds a ton, as is still the standard of the United States government in collecting duties at the custom-houses.

NOTE 3. — The term avoirdupois is from the French *avoir du poid*, signifying to have weight.

NOTE 4. — 1 pound Avoirdupois = 7000 gr. Troy = 1lb. 2oz. 11 pwt. 16 gr. Troy ; 1lb. Troy, or Apothecary = 5760gr. Troy = 13oz. $2\frac{11}{175}$ dr. Avoirdupois ; 1oz. Troy, or Apoth. = 480gr. Troy = 1oz. $1\frac{97}{175}$ dr. Av. ; 1oz. Av. = $437\frac{1}{2}$gr. Troy = 18pwt. $5\frac{1}{2}$gr. Troy ; 1dr. Apoth. = 60gr. Troy = $2\frac{34}{175}$dr. Av. ; 1dr. Av. = $27\frac{11}{22}$gr. Troy = 1pwt. $3\frac{11}{32}$gr. Troy ; 1 pwt. Troy = 24gr. Troy = $\frac{768}{875}$ of a dr. Av. ; 1sc. Apoth. = 20gr. Troy = $\frac{128}{175}$ of a dr. Av.

MENTAL EXERCISES.

1. How many drams in 3oz.? In 7oz.? In 10oz.? In 12oz.?
2. How many ounces in 10lb.? In 15lb.? In 12lb.? In 100lb.?
3. How many pounds in 2 quarters? In 3qr.? In 20qr.?
4. How many quarters in 10cwt.? In 16cwt.? In 17cwt.?
5. How many tons in 80cwt.? In 100cwt.? In 600cwt.?
6. How many hundred weight in 16qr.? In 48qr.? In 96qr.?

EXERCISES FOR THE SLATE.

1. How many pounds in 176T. 17cwt. 3qr. 15lb.?

OPERATION.

176T. 17cwt. 3qr. 15lb.
20
3537 hundred weight.
4
14151 quarters.
25
70770
28302
Ans. 353790 pounds.

2. In 353790lb. how many tons?

OPERATION.

25)353790 lb.
4)14151 qr. 15lb.
20)3537 cwt. 3qr.
176T. 17cwt.
Ans. 176T. 17cwt. 3qr. 15lb.

QUESTIONS. — How many pounds are now allowed for a cwt., and how many for a quarter of a cwt., in most of the United States, in buying and selling articles by weight? How many at the custom-houses? How do you reduce tons to drams? Give the reason for the operation. How do you reduce drams to tons? What is the reason for the operation?

3. In 16T. 19cwt. 0qr. 10lb. 11oz. 5dr. how many drams?
4. In 8681141 drams how many tons?
5. In 679cwt. how many pounds?
6. In 67900lb. how many cwt.?
7. What cost 17cwt. 3qr. 18lb. of beef, at 7 cents per pound?
8. What cost 48T. 17cwt. of lead, at 8 cents per pound?

CLOTH MEASURE.

ART. **90.** Cloth Measure is used in measuring cloth, ribbons, lace, and other articles sold by the yard or ell.

TABLE.

2¼ Inches (in.)	make	1 Nail,	na.
4 Nails	"	1 Quarter of a yard,	qr.
4 Quarters	"	1 Yard,	yd.
3 Quarters	"	1 Ell Flemish,	E. F.
5 Quarters	"	1 Ell English,	E. E.

										na.				in.
								qr.		1	=			2¼
				E. F.				1	=	4	=			9
		yd.		1	=			3	=	12	=			27
E. E.		1	=	1¼	=			4	=	16	=			36
1	=	1¼	=	1⅝	=			5	=	20	=			45

NOTE. — The Ell French is 6 quarters; the Ell Scotch, 4qr. 1⅕in.

MENTAL EXERCISES.

1. In 2 quarters how many nails? In 5qr.? In 8qr.? In 20qr.? In 25qr.? In 30qr.? In 40qr.?
2. In 3 yards how many quarters? In 7yd.? In 8yd.? In 14yd.? In 19yd.? In 100yd.? In 200yd.?
3. How many quarters in 8 nails? In 20na.? In 48na.?
4. How many yards in 20 quarters? In 40qr.? In 100qr.?

EXERCISES FOR THE SLATE.

1. How many nails in 47yd. 3qr. 1na.?
2. In 765 nails how many yards?

QUESTIONS. — Art. 90. For what is cloth measure used? Repeat the table. Is the ell French longer or shorter than the ell English? What makes an ell Scotch?

OPERATION.

4 7 yd. 3qr. 1na.
4
1 9 1 quarters.
4
Ans. 7 6 5 nails.

OPERATION.

4) 7 6 5 na.
4) 1 9 1 qr. 1na.
Ans. 4 7 yd. 3qr. 1na.

3. In 144yd. 3qr. how many quarters?
4. In 579 quarters how many yards?
5. In 17E. E. 4qr. 3na. how many nails?
6. In 359 nails how many ells English?
7. In 126yd. 0qr. 3na. how many nails?
8. In 2019 nails how many yards?
9. What cost 49yd. 3qr. of cloth, at $2.17 per quarter of a yard?
10. What cost 144yd. 1qr. 3na. of cloth, at 25 cents per nail?

LONG MEASURE.

ART. **91.** Long Measure is used in measuring distances in any direction.

TABLE.

12	Inches (in.)	make	1 Foot, ft.
3	Feet	"	1 Yard, yd.
5½	Yards, or 16½ Feet,	"	1 Rod, or Pole, rd.
40	Rods	"	1 Furlong, fur.
8	Furlongs, or 320 Rods,	"	1 Mile, m.
3	Miles	"	1 League, lea.
69⅛	Miles (nearly)	"	1 Degree, deg. or °.
360	Degrees	"	1 Circle of the Earth.

														ft.						in.
												yd.		1	=					12
					rd.							1	=	3	=					36
		fur.			1	=				5½	=	16½	=	198						
m.		1	=		40	=				220	=	660	=	7920						
1	=	8	=		320	=				1760	=	5280	=	63360						

QUESTIONS. — How do you reduce yards to nails? How do you reduce nails to yards? What is the reason for the operation? How is long measure used? Repeat the table.

NOTE. — 1. 12 lines make 1 inch ; 4 inches, 1 hand ; 6 feet, 1 fathom ; $\frac{1}{60}$ of a degree of the circumference of the earth, 1 knot, or geographical mile, equal to $1\frac{1}{2}$ statute miles.

NOTE. — 2. The *yard* is the *standard unit* of linear measure adopted by the United States government, and it is the same as the imperial yard of Great Britain. A *metre*, the unit of linear measure, as established by the French government, is equal to about $39\frac{37}{100}$ English inches.

NOTE. — 3. The English statute mile is the same as that of the United States, but that of other countries differs in value from it ; as the German short mile is equal to 6857 yards, or about $3\frac{9}{10}$ English miles ; the German long mile, to 10125 yards, or about $5\frac{3}{4}$ English miles ; the Prussian mile, to 8237 yards, or about $4\frac{7}{10}$ English miles ; the Spanish common league, to 7416 yards, or about $4\frac{1}{5}$ English miles ; the Spanish judicial league, to 4635 yards, or about $2\frac{2}{5}$ English miles.

NOTE. — 4. A degree of longitude is $\frac{1}{360}$ of any circle of latitude. As the circles of latitude diminish in length, the degrees of longitude vary in length under different parallels of latitude. Thus, under the equator, the length of a degree of longitude is about $69\frac{1}{6}$ statute miles ; at 25° of latitude, $62\frac{7}{10}$ miles ; at 40° of latitude, 53 miles ; at 42° of latitude, $51\frac{1}{4}$ miles ; at 49° of latitude, $45\frac{1}{4}$ miles ; at 60°, $34\frac{7}{12}$ miles.

MENTAL EXERCISES.

1. How many inches in 4 feet? In 10ft.? In 12ft.? In 20ft.?
2. How many feet in 2 yards? In 5yd.? In 20yd.? In 18yd.?
3. How many rods in 2 furlongs? In 8fur.? In 1fur.? In 30 fur.? In 100fur.? In 200fur.? In 400fur.?
4. How many leagues in 9 miles? In 21m.? In 81m.? In 144m.? In 40m.? In 50m.? In 80m.?
5. How many furlongs in 120 rods? In 360rd.? In 1440rd.?
6. How many yards in 99 feet? In 66ft.? In 144ft.?
7. How many feet in 108 inches? In 144in.? In 1728in.?

EXERCISES FOR THE SLATE.

1. In 66deg. 56m. 7fur. 37rd. 12ft. 9in. how many inches?

QUESTIONS. — How many lines make 1 inch? How many inches 1 hand? How many feet 1 fathom? What is the standard unit of linear measure adopted by the United States? Is the value of the mile the same in all countries? How much is a degree of longitude under the equator? At 40° of latitude? At 42° of latitude? At 60° of latitude?

OPERATION.

6 6deg. 56m. 7fur. 37rd. 12ft. 9in.

$69\frac{1}{6}$

600
401
11

4621 miles.
8

36975 fur.
40

1479037 rods.
$16\frac{1}{2}$

8874224
1479038
$739518\frac{1}{2}$

$24404122\frac{1}{2}$ ft.
12

292849479 in. Ans.

2. In 292849479 inches how many degrees?

OPERATION.

12)292849479

$16\frac{1}{2}$)24404123 ft. 3in.
2 2

33)48808246 [12ft. 6in.

40)1479037 rd. 25÷2=

8)36975 fur. 37rd.

$69\frac{1}{6}$)4621 miles 7fur.
6 6

415)27726

66 deg. 336 ÷ [6 = 56m.

66deg. 56m. 7fur. 37rd. $12\frac{1}{2}$ft. 3in.
$\frac{1}{2}$ = 6in.

66 56 7 37 12 9 Ans.

NOTE. — To multiply by $\frac{1}{2}$, we take $\frac{1}{2}$ of the multiplicand. — To divide by $16\frac{1}{2}$, we first reduce both the divisor and dividend to halves, and then divide; and the remainder being 25 half-feet, we take half of it for the true remainder = 12ft. 6in. We adopt the same principle in dividing by $69\frac{1}{6}$; the remainder being 336 sixths of miles, we divide them by 6, and the quotient is 56 miles. By adding the 6 inches to the 3 inches, we obtain the true answer.

3. In 47 miles how many feet?
4. In 248160 feet how many miles?
5. In 78deg. 50m. 7fur. 30rd. 5yd. 2ft. 10in. how many inches?
6. How many degrees in 345056794 inches?

QUESTIONS. — How do you reduce degrees to inches? Give the reason of the operation. How do you reduce inches to degrees? What is the reason for the operation? How do you multiply by $\frac{1}{2}$? How do you divide by $16\frac{1}{2}$ and find the true remainder? How do you obtain the true answer in examples of this kind?

SURVEYORS' MEASURE.

ART. **92.** This measure is used by surveyors in measuring land, roads, &c.

TABLE.

$7\frac{92}{100}$	Inches (in.)						make	1	Link,			l
25	Links						"	1	Pole,			p.
100	Links, 4 Poles, or 66 Feet,						"	1	Chain,			cha.
10	Chains						"	1	Furlong,			fur.
8	Furlongs, or 80 Chains,						"	1	Mile,			m.
								l.				in.
						p.		1	=			$7\frac{92}{100}$
			cha.			1	=	25	=			198
	fur.		1	=		4	=	100	=			792
m.	1	=	10	=		40	=	1000	=			7920
1	= 8	=	80	=		320	=	8000	=			63360

NOTE. — Gunter's chain, in length 4 poles, or 66 feet, and divided into 100 links, is that mostly used in ordinary land surveys; but in locating roads, and like public works, an engineer's chain is usually 100 feet in length, containing 120 links, each 10 inches long.

MENTAL EXERCISES.

1. In 2 poles how many links? In 4 poles? In 7 poles?
2. In 5 chains how many links? In 8cha.? In 10cha.?
3. How many poles in 50 links? In 75l.? In 125l.?

EXERCISES FOR THE SLATE.

1. How many links in 7m. 5fur. 6cha. 30l.?

OPERATION.

7 m. 5fur. 6cha. 30l.
8
61 furlongs.
10
616 chains.
100
61630 links. Ans.

2. In 61630 links how many miles?

OPERATION.

100)61630 l.
10)616 cha. 30l.
8)61 fur. 6cha.
7 m. 5fur.

Ans. 7m. 5fur. 6cha. 30l.

3. How many miles in 4386 chains?

QUESTIONS. — Art. 92. For what is surveyors' measure used? Recite the table. How do you reduce miles to links? What is the reason for the operation? How do you reduce inches to chains? To miles? Give the reason of the operation.

4. In 54m. 66cha. how many chains?
5. In 75m. 49cha. how many poles?
6. How many miles in 24196 poles?
7. How many links in 7m. 4fur. 30rd.?
8. How many miles in 60750 links?

SQUARE MEASURE.

ART. **93.** Square Measure is used in measuring surfaces of all kinds.

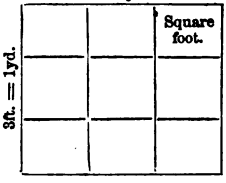
TABLE.

144	Square inches	make	1 Square foot,	ft.
9	Square feet	"	1 Square yard,	yd.
30¼	Square yards, or	"	1 Square rod or pole,	p.
272¼	Square feet,			
40	Square rods or poles	"	1 Rood,	R.
4	Roods, or 160 Poles,	"	1 Acre,	A.
640	Acres	"	1 Square mile,	S. M.

					ft.	in.
				yd.	1 =	144
			p.	1 =	9 =	1296
		R.	1 =	30¼ =	272¼ =	39204
	A.	1 =	40 =	1210 =	10890 =	1568160
S. M.	1 =	4 =	160 =	4840 =	43560 =	6272640
1 =	640 =	2560 =	102400 =	3097600 =	27878400 =	4014489600

NOTE. — A *square* is a figure bounded by four equal lines, perpendicular to each other.

When the four lines are each 1 foot in length, the space enclosed is 1 *square foot*; when 1 yard in length, 1 *square yard*; when 1 rod in length, 1 *square rod*; and so for any other dimension.



In this diagram the *large* square represents a square *yard*, and each of the *smaller* squares within it represents one square *foot*. Now, since there are *three* rows of small squares, and *three* square feet in each row, there will be 3 sq. ft. × 3 = 9 sq. ft. in the large square. But the large square is 3 ft. in length, and 3 ft. in breadth; hence,

To find the contents of a square, multiply the numbers denoting its length and breadth together.

QUESTIONS. — Art. 93. For what is square measure used? Repeat the table. What is a square? What is a square foot? How may the contents of a square be found? Explain by the diagram the reason of the operation.

MENTAL EXERCISES.

1. In 2 square feet how many square inches?
2. In 3 square yards how many square feet? In 10 sq. yd.?
3. In 5 roods how many poles? In 20 roods? In 30 roods?
4. In 7 acres how many roods? In 24 acres? In 40 acres?

EXERCISES FOR THE SLATE.

1. How many square inches in 12A. 3R. 24p. 144ft. 72in.?

OPERATION.

12 A. 3R. 24p. 144ft. 72in.
4
51 roods.
40
2064 poles.
272 1/4
4132
14452
4129
516
562068 feet.
144
2248274
2248279
562068

Ans. 80937864 inches.

NOTE. — To multiply by 1/4, we take 1/4 of the multiplicand.

2. In 80937864 square inches how many inches?

OPERATION.

144) 80937864 inches.
272 1/4) 562068 ft. 72in.
4 4
1089) 2248272 fourths of a foot.
40) 2064 poles. 576 ÷ 4 = 144ft.
4) 51 R. 24p.
Ans. 12 A. 3R. 24p. 144ft. 72in.

NOTE. — To divide by the 272 1/4, we first reduce the divisor and dividend to *fourths*, and then divide. The remainder obtained, being *fourths*, is reduced to whole numbers by dividing by 4.

QUESTIONS. — How do you reduce acres to square inches? Give the reason for the operation. How do you reduce square inches to acres? What is the reason for the operation? How do you multiply by 1/4?

3. In 49A. 3R. 16p. how many square feet?

4. In 2171466 square feet how many acres?

5. What is the value of 365A. 3R. 17p. at $1.75 per square rod or pole?

6. Sold a valuable piece of land, containing 3A. 1R. 30p., at $1.25 per square foot; what was received for the land?

7. In a tract of land 12 miles square, how many square miles? How many acres?

8. In 18A. 0R. 16p. how many square feet?

9. Purchased 48A. 3R. 14p. of land for $2.25 per square rod, and sold the same for $3.15 per square rod; what did I gain by my bargain?

CUBIC OR SOLID MEASURE.

ART. **94.** Cubic or Solid Measure is used in measuring such bodies or things as have length, breadth, and thickness; as timber, stone, &c.

TABLE.

1728	Cubic inches (cu. in.)	make	1 Cubic foot,	cu. ft.
27	" feet	"	1 " yard,	cu. yd.
40	" feet	"	1 Ton,	T.
16	" feet	"	1 Cord foot,	c. ft.
8 Cord feet, or 128 Cubic feet,		"	1 Cord of wood,	C.

					yd.		ft.			in.
					1		1	=		1728
		T.			1	=	27	=		46656
C		1	=		$1\frac{13}{27}$	=	40	=		69120
1	=	$3\frac{1}{5}$	=		$4\frac{23}{27}$	=	128	=		221184

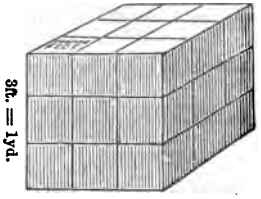
NOTE 1. — A pile of wood 8ft. in length, 4ft. in breadth, and 4ft. in height, contains a cord.

Also, one ton of timber, as usually surveyed, contains $50\frac{92}{100}$ cubic or solid feet.

QUESTIONS: — How do you divide by 2721? Of what denomination is the remainder? How is the true remainder found? — Art. 94. For what is cubic measure used? Recite the table. What are the dimensions of a pile of wood containing 1 cord? How many solid feet does a ton of timber contain, as usually surveyed?

NOTE 2. — A *cube* is a solid bounded by six square and equal sides.

If the cube is 1 foot long, 1 foot wide, and 1 foot high, it is called a *cubic* or *solid foot*. If the cube is 3 feet long, 3 feet wide, and 3 feet thick, it is called a *cubic* or *solid yard*.



Now, since each side of a cubic yard, as represented in the diagram, contains 9 sq. ft. of surface (Art. 93), it is plain, if a block be cut off from one side, *one* foot thick, it can be divided into 9 solid blocks, with sides 1 foot in length, breadth, and thickness, and therefore will contain 9 solid feet; and since the whole block or cube is *three* feet thick, it must contain 9 solid feet $\times$ 3 = 27 solid feet; or 3 solid feet $\times$ 3 $\times$ 3 = 27 solid feet. Hence,

To find the contents of a cubic body, multiply together the numbers denoting the length, breadth, and thickness.

MENTAL EXERCISES.

1. In 2 cubic feet how many cubic inches? In 4 cu. ft.?
2. In 3 cubic yards how many cubic feet? In 10 cu. yd.?
3. How many cords of wood in 64 cord feet? In 96 c. ft.?
4. How many tons in 80 cu. ft. of timber? In 160 cu. ft.?

EXERCISES FOR THE SLATE.

1. In 48 cu. yd. and 15 cu. ft. how many cubic inches?

OPERATION.

48 yd. 15ft.
27
341
97
1311 feet.
1728
10488
2622
9177
1311
Ans. 2265408 inches.

2. In 2265408 cubic inches how many cubic yards?

OPERATION.

1728)2265408 cu. in.
27)1311 cu. ft.
Ans. 48 yd. 15ft.

QUESTIONS. — What is a cube? How do you find the contents of a cube? Give the reason for the operation. Describe a cubic foot. How do you reduce a ton to cubic inches? Give the reason for the operation. How do you reduce cubic inches to cubic yards? Give the reason for the operation.

3. In 45 cords of wood how many cubic inches?

4. In 9953280 cubic inches how many cords of wood?

5. How many cubic feet in a pile of wood 15ft. long, 4ft. wide, and $6\frac{1}{2}$ft. high? How many cords?

6. How many cubic inches in a block of marble 4ft. long, $3\frac{1}{4}$ft. wide, and 2ft. thick?

7. In a room 14ft. long, 12ft. wide, and 8ft. high, how many cubic feet?

8. What will 9080 cubic feet of ship-timber cost, at $11.50 per ton?

WINE OR LIQUID MEASURE.

ART. **95.** Wine or Liquid Measure is used in measuring all kinds of liquids, except, in some places, beer, ale, porter, and milk.

TABLE.

4 Gills (gi.)	make	1 Pint,	pt.
2 Pints	"	1 Quart,	qt.
4 Quarts	"	1 Gallon,	gal.
63 Gallons	"	1 Hogshead,	hhd.
2 Hogsheads	"	1 Pipe, or Butt,	pi.
2 Pipes	"	1 Tun,	tun.

tun.		pi.		hhd.		gal.		qt.		pt.		gi.
										1	=	4
								1	=	2	=	8
						1	=	4	=	8	=	32
				1	=	63	=	252	=	504	=	2016
		1	=	2	=	126	=	504	=	1008	=	4032
1	=	2	=	4	=	252	=	1008	=	2016	=	8064

NOTE 1.—By laws of Massachusetts, 32 gallons make 1 *barrel*. In some states $31\frac{1}{2}$ gallons, and in others from 28 to 32 gallons, make 1 barrel. 42 gallons make 1 *tierce*, and 2 tierces, 1 *puncheon*.

NOTE 2.—The term hogshead is often applied to any large cask that may contain from 50 to 120 gallons, or more.

NOTE 3.—The *Standard Unit* of *Liquid Measure* adopted by the government of the United States is the *Winchester Wine Gallon*, which contains 231 cubic inches. It has the name Winchester, from its standard having been formerly kept at Winchester, England. The *Imperial Gallon*, now adopted in Great Britain, contains $277\frac{274}{1000}$ cubic inches; so that 6 Winchester gallons make about 5 Imperial gallons.

QUESTIONS.—Art. 95. For what is wine or liquid measure used? Repeat the table. How many gallons make a barrel? How many a tierce? How many a puncheon? How is the term hogshead often applied? What is the standard unit of liquid measure?

MENTAL EXERCISES.

1. In 3 pints how many gills? In 5 pints? In 9 pints?
2. In 4 quarts how many pints? In 6 quarts? In 8 quarts?
3. In 5 gallons how many quarts? In 7 gallons?
4. How many gallons in 12 quarts? In 18 quarts?

EXERCISES FOR THE SLATE.

1. In 47 tuns of wine how many gills?

OPERATION.

47 tuns.
4
188 hogsheads.
63
564
1128
11844 gallons.
4
47376 quarts.
2
94752 pints.
4
Ans. 379008 gills.

2. In 379008 gills how many tuns?

OPERATION.

4)379008 gi.
2)94752 pt.
4)47376 qt.
63)11844 gal.
4)188 hhd.
Ans. 47 tuns.

3. Reduce 197 tuns 3hhd. 60gal. 3qu. 1pt. to gills.
4. In 1596604 gills how many tuns?
5. What will 7 hogsheads of wine cost, at 5 cents a pint?
6. What cost 18 tuns 1hhd. 47gal. of oil, at $1.25 per gallon?

BEER MEASURE.

ART. **96.** Beer Measure is used in measuring beer, ale, porter, and milk.

TABLE.

2 Pints (pt.)		make		1 Quart,		qt.
4 Quarts		"		1 Gallon,		gal.
54 Gallons		"		1 Hogshead,		hhd.
				qt.		pt.
		gal.		1	=	2
hhd.		1	=	4	=	8
1	=	54	=	216	=	432

QUESTION.—Art. 96. Repeat the table of beer measure.

NOTE 1. — The gallon of beer measure contains 282 cubic inches; and as has been usually reckoned, 36 gallons equal 1 barrel; 2 hogsheads, or 108 gallons, 1 butt; 2 butts, or 216 gallons, 1 tun. 1 gallon beer measure = 1gall. 1pt. $3\frac{5}{7}$ gi. wine measure.

NOTE 2. — Beer Measure is becoming obsolete. Milk and malt liquors, at the present time, are bought and sold, very generally, by wine or liquid measure.

EXERCISES FOR THE SLATE.

1. How many quarts in 76 hogsheads?

OPERATION.

76 hhd.
54
304
380
4104 gallons.
4
16416 quarts.

Ans. 16416 quarts.

2. In 16416 quarts how many hogsheads?

OPERATION.

4)16416 qt.
54)4104 gal.
Ans. 76 hhd.

3. In 4 tuns 1hhd. 17gal. 1pt. how many pints?
4. How many tuns in 7481 pints?
5. What cost 7hhd. 18gal. of beer at 4 cents a quart?
6. At 15 cents per gallon, what will 18hhds. of ale cost?

DRY MEASURE.

ART. 97. This measure is used in measuring grain, fruit, salt, coal, &c.

TABLE.

2 Pints (pt.)	make	1 Quart,	qt.
8 Quarts	"	1 Peck,	pk.
4 Pecks	"	1 Bushel,	bu.
8 Bushels	"	1 Quarter,	qr.
36 Bushels	"	1 Chaldron,	ch.

						qt.		pt.
				pk.		1	=	2
		bu.		1	=	8	=	16
ch.		1	=	4	=	32	=	64
1	=	36	=	144	=	1152	=	2304

QUESTIONS. — How many cubic inches does the beer gallon contain? How do you reduce hogsheads to quarts? How do you reduce quarts to hogsheads? — *Art. 97. For* what is dry measure used? Repeat the table.

NOTE 1. — The *Standard Unit* of *Dry Measure* adopted by the United States government is the *Winchester* bushel, which is in form a cylinder, 18½ inches in diameter, and 8 inches deep, containing $2150\frac{42}{100}$ cubic inches. The *Standard Imperial* bushel of Great Britain contains $2218\frac{192}{1000}$ cubic inches, so that 32 Imperial bushels equal about 33 Winchester bushels. The gallon in Dry Measure contains $268\frac{4}{5}$ cubic inches.

NOTE 2. — 1gal. Dry Measure = $268\frac{4}{5}$cu. in. = 1gal. 1pt. $1\frac{3}{5}$gi. Wine Measure = 3qt. $1\frac{1}{2}\frac{4}{3}\frac{7}{5}$pt. Beer Measure; 1gal. W. M. = 231cu. in. = 3 qt. $\frac{1}{8}$pt. D. M. = 3qt. $\frac{28}{4}\frac{}{7}$ pt. B. M.; 1gal. B. M. = 282cu. in. = 1gal. 1pt. $3\frac{5}{7}$gi. W. M. = 1gal. $\frac{1}{2}\frac{1}{8}$pt. D. M.; 1qt. D. M. = $67\frac{1}{5}$cu. in. = 1qt. $1\frac{1}{5}\frac{7}{5}$gi. W. M.; 1qt. W. M. = $57\frac{3}{4}$cu. in. = $1\frac{2}{3}\frac{3}{2}$pt. D. M.; 1pt. D. M. = $33\frac{3}{5}$cu. in. = 1pt. $\frac{1}{5}\frac{7}{5}$ gi. W. M.; 1pt. W. M. = $28\frac{7}{8}$cu. in. = $\frac{6}{6}\frac{5}{4}$pt. D. M.

MENTAL EXERCISES.

1. In 2 quarts how many pints? In 5 quarts? In 7 quarts?
2. In 3 pecks how many quarts? In 6 pecks? In 9 pecks?
3. In 5 bushels how many pecks? In 10 bushels?
4. How many pecks in 16 quarts? In 25 quarts?

EXERCISES FOR THE SLATE.

1. How many quarts in 49ch. 8bu. 3pk. and 3qt.?

OPERATION.

49 ch. 8bu. 3pk. 3qt.
36
302
147
1772 bushels.
4
7091 pecks.
8
Ans. 56731 quarts.

2. In 56731 quarts how many chaldrons?

OPERATION.

8)56731 qt.
4)7091 pk. 3qt.
36)1772 bu. 3pk.
49 ch. 8bu.

Ans. 49ch. 8bu. 3pk. 3qt

3. Reduce 97ch. 30bu. 2pk. to quarts.
4. In 112720 quarts how many chaldrons?
5. How many pints in 35bu. 1pt.?
6. Reduce 2241 pints to bushels.
7. Reduce 18qr. 3pk. 5qt. to quarts.
8. How many quarters in 4637 quarts?
9. In 19bu. 3pk. 7qt. 1pt. how many pints?
10. In 1279 pints how many bushels?

QUESTION. — What is the standard unit of Dry Measure?

MEASURE OF TIME.

ART. 98. This measure is applied to the various divisions and subdivisions into which time is divided.

TABLE.

60 Seconds (sec.)	make	1 Minute,	m.
60 Minutes	"	1 Hour,	h.
24 Hours	"	1 Day,	da.
7 Days	"	1 Week,	w.
365¼ Days, or 52 weeks 1¼ days,	"	1 Julian Year,	y.
12 Calendar Months (mo.)	"	1 Year,	y.

y.		w.		d.		h.		m.		sec.
								1	=	60
						1	=	60	=	3600
				1	=	24	=	1440	=	86400
		1	=	7	=	168	=	10080	=	604800
1	=	$52\frac{5}{28}$	=	365¼	=	8766	=	525960	=	31557600

NOTE 1. — The true *Solar* or *Tropical Year* is the time measured from the sun's leaving either equinox or solstice to its return to the same again, and is 365d. 5h. 48m. 49sec. nearly.

The *Julian Year*, so called from the calendar instituted by Julius Cæsar, contains 365¼ days, as a medium; three years in succession containing 365 days, and the fourth year 366 days; which, as compared with the true solar year, produces an average yearly error of 11m. $10\frac{2}{10}$ sec., or a difference that would amount to 1 whole day in about 120 years.

The *Gregorian Year*, or that instituted by Pope Gregory XIII., in the year 1582, and which is now the *Civil* or *Legal Year* in use among the different nations of the earth, contains, like the Julian year, 365 days for three years in succession, and 366 days for the fourth, *excepting centennial years whose number cannot be exactly divided by* 400. The Gregorian year is so nearly correct as to err only 1 day in 3866 years, a difference so little as hardly to be worth taking into account.

A *Common Year* is one of 365 days, and a *Leap* or *Bissextile Year* is one of 366 days. Any year is Leap Year whose number can be divided by 4 without a remainder, except years whose number can be divided without a remainder by 100, but not by 400.

A *Sidereal Year* is the time in which the earth revolves round the sun, and is 365d. 6h. 9m. $9\frac{6}{10}$ sec.

NOTE 2. — The names of the 12 calendar months, composing the civil year, are January, February, March, April, May, June, July, August, September, October, November, December, and the number of days in each may be readily remembered by the following lines:

"Thirty days hath September,
April, June, and November;

QUESTIONS. — Art. 98. To what is the measure of time applied? Repeat the table. How is the true solar year measured? How long is it? Why is the Julian year so called? Who instituted the Gregorian year? What is a Common year? What is a Sidereal year?

And all the rest have thirty-one,
Save February, which alone
Hath twenty-eight; and this, in fine,
One year in four hath twenty-nine."

TABLE.

SHOWING THE NUMBER OF DAYS FROM ANY DAY OF ONE MONTH TO THE SAME DAY OF ANY OTHER MONTH IN THE SAME YEAR.

FROM ANY DAY OF	TO THE SAME DAY OF											
	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
January	365	31	59	90	120	151	181	212	243	273	304	334
February	334	365	28	59	89	120	150	181	212	242	273	303
March	306	337	365	31	61	92	122	153	184	214	245	275
April	275	306	334	365	30	61	91	122	153	183	214	244
May	245	267	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
August	153	184	212	243	273	304	334	365	31	61	92	122
September	122	153	181	212	242	273	303	334	365	30	61	91
October	92	123	151	182	212	243	273	304	335	365	31	61
November	61	92	120	151	181	212	242	273	304	334	365	30
December	31	62	90	121	151	182	212	243	274	304	335	365

For example, suppose we wish to find the number of days from April 4th to November 4th, we look for April in the left-hand vertical column, and November at the top, and, where the lines intersect, is 214, the number sought. Again, if we wish the number of days from June 10th to September 16th, we find the difference between June 10th and September 10th to be 92 days, and add 6 days for the excess of the 16th over the 10th of September, so we have 98 days as the exact difference.

If the end of February be included between the points of time, a day must be added in leap year.

When the time includes more than one year, there must be added 365 days for each year.

MENTAL EXERCISES.

1. In 3 minutes how many seconds? In 5 minutes?
2. In 2 hours how many minutes? In 4 hours?
3. In 4 weeks how many days? In 6 weeks? In 9 weeks?
4. In 2 days how many hours? In 3 days? In 7 days?
5. How many weeks in 21 days? In 30 days? In 50 days?
6. How many calendar months in 2 years? In 8 years? In 10 years? In 12 years? In 20 years?

QUESTIONS. — Name the months in their order. How many days has each month? How do you find by the table the number of days from April 4th to November 4th? When the time sought for is more than one year, how many days must be added?

Exercises for the Slate.

1. How many seconds in 365da. 2. In 31556929 seconds 5h. 48m. 49sec., or one solar year? how many days?

OPERATION.

365 da. 5h. 48m. 49sec.

24

1465
730

8765 hours.
60

525948 minutes.
60

Ans. 31556929 seconds.

OPERATION.

60) 31556929

60) 525948 m. 49sec.

24) 8765 h. 48m.

365 da. 5h.

Ans. 365da. 5h. 48m. 49sec.

3. Reduce 296da. 18h. 32m. to minutes.
4. In 427352 minutes how many days?
5. How many seconds in 30 solar years 262da. 17h. 28m. 42sec.?
6. In 969407592 seconds how many solar years?
7. How many weeks in 684592 minutes?
8. In 67w. 6d. 9h. 52m. how many minutes?
9. How many days from June 5th to Dec. 11th?
10. How many days from March 17th, 1856, to May 16th, 1857?
11. How many days from December 18th, 1856, to January 30th, 1857?
12. How many days from August 30th, 1857, to June 1st, 1858?
13. How many days from July 4th, 1859, to July 4th, 1860?
14. How many days from April 25th, 1855, to August 20th, 1858?

Note. — The last six examples are to be performed by aid of the table on page 103.

Questions. — How do you reduce years to seconds? Give the reason for the operation. How do you reduce seconds to days? To years? Give the reason for the operation.

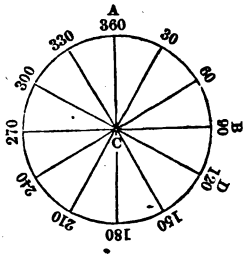
CIRCULAR MEASURE.

ART. **99.** Circular Measure is applied to the measurement of circles and angles, and is used in reckoning latitude and longitude, and the revolutions of the planets round the sun.

TABLE.

60 Seconds (″)	make	1 Minute, ′.
60 Minutes	"	1 Degree, °.
30 Degrees	"	1 Sign, S.
12 Signs, or 360 Degrees,	"	The Circle of the Zodiac, C.

					°		′		″
					1	=	1	=	60
			S.		1	=	60	=	3600
C.			1	=	30	=	1800	=	108000
1	=	12	=		360	=	21600	=	1296000



NOTE. — 1. A *Circle* is a plane figure bounded by a curve line, every part of which is equally distant from a point called its centre.

The *Circumference* of a circle is the line which bounds it, as shown by the diagram.

An *Arc* of a circle is any part of its circumference ; as AB.

A *Radius* of a circle is a straight line drawn from its centre to its circumference; as CA, CB, or CD.

Every circle, large or small, is supposed to be divided into 360 equal parts, called degrees.

A *Quadrant* is one fourth of a circle, or an arc of 90° ; as AB.

An *Angle*, as ACB, is the inclination or opening of two lines which meet at a point, as C. The point is the *vertex* of the angle. If a circle be drawn around the vertex of an angle as a centre, the two sides of the angle, as *radii* of the circle, will include an arc, which is the *measure* of the angle ; as the arc AD = 120° is the measure of the angle ACD, and AB = 90°, the measure of the angle ACB ; hence the one is called an angle of 120°, and the other an angle of 90°.

NOTE. — 2. As the earth turns on its axis from west to east every 24 hours, the sun appears to pass from east to west $\frac{1}{24}$ of 360° of longitude every hour, or over 15° of longitude in 1 hour's time, or 1° in 4 minutes of time, and 1′ in 4 seconds of time ; so that, for instance, at any place, when it is noon, it is 1 hour earlier for every 15° of longitude westward, or 1 hour later for every 15° of longitude eastward. Thus, Boston being 71° 4′ west of Greenwich, and San Francisco 51° 17′ west of Boston, when it is noon at Boston, it is 4h. 44m. 16sec. past noon at Greenwich, and wanting 3h. 25m. 8sec. of noon at San Francisco.

QUESTIONS. — Art. 99. To what is circular measure applied? Recite the table. What is a circle? What is an angle?

Exercises for the Slate.

1. How many minutes in 11S. 18° 57′?

OPERATION.

11S. 18° 57′
30
348 degrees.
60
Ans. 20937 minutes.

2. In 20937 minutes how many signs?

OPERATION.

60)20937′
30)348° 57′
11S. 18°.

Ans. 11S. 18° 57′.

3. In 27S. 19° 51′ 28″ how many seconds?
4. How many signs in 2987488 seconds?

MISCELLANEOUS TABLE.

Art. **100.** This table embraces a variety of things in business important to be known.

12 units	make	1 dozen.
12 dozen	"	1 gross.
12 gross	"	1 great gross.
20 units	"	1 score.
14 pounds of Iron or Lead	"	1 stone.
60 pounds of Wheat	"	1 bushel.
60 pounds of Clover-seed	"	1 bushel.
60 pounds of Beans	"	1 bushel.
60 pounds of Potatoes	"	1 bushel.
52 pounds of Onions	"	1 bushel.
70 pounds of Corn on the Cob	"	1 bushel.
56 pounds of Shelled Corn	"	1 bushel.
56 pounds of Rye	"	1 bushel.
56 pounds of Flax-seed	"	1 bushel.
45 pounds of Timothy-seed	"	1 bushel.
20 pounds of Bran	"	1 bushel.
48 pounds of Barley	"	1 bushel.
52 pounds of Buckwheat	"	1 bushel in Ky.
48 pounds of Buckwheat	"	1 bushel in Mass. and Pa.
32 pounds of Oats	"	1 bushel in Ms., Ill., O., &c.
30 pounds of Oats	"	1 bushel in Me., N.H., Pa., [&c.

Questions. — How do you reduce signs to seconds? Give the reason of the operation. How do you reduce seconds to degrees? To signs? Give the reason for the operation. How many degrees in a circle? — Art. 100. What *is embraced in* the miscellaneous table?

196 pounds of Flour	make	1 barrel.
200 pounds of Beef	"	1 barrel.
200 pounds of Pork	"	1 barrel.
100 pounds of Fish	"	1 quintal.
200 pounds of Shad or Salmon	"	1 barrel in N. Y., Ct.
220 pounds of Fish	"	1 barrel in Md.
30 gallons of Fish	"	1 barrel in Mass.
5 bushels of Corn	"	1 barrel in Md., Tenn., &c.

OF BOOKS.

A sheet folded in	2	leaves	forms a	folio.
A sheet "	4	leaves	"	a quarto.
A sheet "	8	leaves	"	an octavo.
A sheet "	12	leaves	"	a 12mo.
A sheet "	18	leaves	"	an 18mo.
A sheet "	24	leaves	"	a 24mo.

MISCELLANEOUS EXERCISES IN REDUCTION.

1. In $345.18 how many mills?
2. How many dollars in 345180 mills?
3. In 46£ 18s. 5d. how many farthings?
4. How many pounds in 45044 farthings?
5. Reduce 61lb. 0oz. 17pwt. 17gr. troy to grains.
6. In 351785 grains troy how many pounds?
7. How many scruples in 27℔ 3℥ 1℥ 1℈?
8. In 7852 scruples how many pounds?
9. In 83T. 11cwt. 3qr. 18lb. how many ounces?
10. How many tons in 2675088 ounces?
11. How many nails in 97yd. 3qr. 3na.?
12. In 1567 nails how many yards?
13. In 57 ells English how many yards?
14. How many ells English in 71yd. 1qr.?
15. How many inches in 15m. 7fur. 18rd. 10ft. 6in.?
16. In 1009530 inches how many miles?
17. In 95,000,000 of miles how many inches?
18. How many miles in 6,019,200,000,000 inches?
19. In 48deg. 18m. 7fur. 18rd. how many feet?
20. In 17629557 feet how many degrees?
21. How many square feet in 7A. 3R. 16p. 218ft.?
22. In 342164 square feet how many acres?
23. How many square inches in 25 square miles?

QUESTION.—What gives name to the size or form of books?

24. In 100362240000 square inches how many square miles?
25. How many cubic inches in 15 tons of timber?
26. In 1036800 cubic inches how many tons?
27. How many gills of wine in 5hhd. 17gal. 3qt.?
28. In 10648 gills how many hogsheads of wine?
29. How many quarts of beer in 29hhd. 30gal. 3qt.?
30. In 6387 quarts of beer how many hogsheads?
31. How many pints in 15ch. 16bu. 3pk. of wheat?
32. In 35632 pints of wheat how many chaldrons?
33. How many seconds of time in 365 days 6 hours?
34. In 31557600 seconds how many days?
35. How many hours in 1842 years (of 365da. 6h. each)?
36. In 16146972 hours how many years?
37. How many seconds in 8S. 14° 18′ 17″?
38. In 915497″ how many signs?
39. What will be the cost of 13 gross of steel pens, at $2\frac{1}{2}$ cents per pen?
40. Bought 12 reams of paper at 20 cents per quire; how much did it cost?
41. I wish to put 2 hogsheads of wine into bottles that will contain 3 quarts each; how many bottles are required?

42. When $1480 are paid for 25 acres of land, what costs 1 acre? What costs 1 rood? What cost 37A. 2R. 18p.?

43. John Webster bought 5cwt. 3qr. 18lb. of sugar at 9 cents per lb., for which he paid 25 barrels of apples at $1.75 per barrel; how much remains due?
44. Bought a silver tankard weighing 2lb. 7oz. for $46.50; what did it cost per oz.? How much per lb.?
45. Bought 3T. 1cwt. 18lb. of leather at 12 cents per lb., and sold it at 9 cents per lb.; what did I lose?

46. Phineas Bailey has agreed to grade a certain railroad at $5.75 per rod; what will he receive for grading a road between two cities, whose distance from each other is 37m. 7fur. 29rd.?

47. If it cost $17.29 per rod to grade a certain piece of railroad, what will be the expense of grading 15m. 6fur. 37rd.?

48. What is the value of a house-lot, containing 40 square rods and 200 square feet, at $1.50 per square foot?

49. How many yards of carpeting, that is one yard in width, will be required to carpet a room 18ft. long and 15ft. wide?

50. A certain machine will cut 120 shingle-nails in a minute; how many will it cut in 47 days 7 hours, admitting the machine to be in operation 10 hours per day?

51. In a field 80 rods long and 50 rods wide, how many square rods? How many acres?

52. How long will it take to count 18 millions, counting at the rate of 90 a minute?

53. A merchant purchased 9 bales of cloth, each containing 15 pieces, each piece 23 yards, at 8 cents per yard; what was the amount paid?

54. Suppose a certain township is 6 miles long and $4\frac{1}{2}$ miles wide, how many lots of land of 90 acres each does it contain?

55. The pendulum of a certain clock vibrates 47 times in 1 minute; how many times will it vibrate in 196 days 49m.?

56. How many shingles will it take to cover a roof, each of whose equal sides is 36 feet long, with rafters 16 feet in length, supposing 1 shingle to cover 27 square inches?

57. How many times will the large wheels of an engine turn round in going from Boston to Portland, a distance of 110 miles, supposing the wheels to be 12 feet and 6 inches in circumference?

58. In a certain house there are 25 rooms, in each room 7 bureaus, in each bureau 5 drawers, in each drawer 12 boxes, in each box 15 purses, in each purse 178 sovereigns, each sovereign valued at $4.84; what is the amount of the money?

59. In 18rd. 5yd. 2ft. 11in. how many inches?

60. In 3779 inches how many rods?

Ans. 18rd. 5yd. 2ft. 11in.

61. Sold 5T. 17cwt. 3qr. 18lb. of potash for 3 cents per pound; what was the amount?

62. A gentleman purchased a house-lot that was 25 rods long and 16 rods wide for $100,000, and sold the same for $1.25 per square foot; what did he gain by his purchase?

§ XI. ADDITION OF COMPOUND NUMBERS.

Art. 101. Addition of Compound Numbers is the process of finding the amount of two or more compound numbers.

ENGLISH MONEY.

Ex. 1. Paid a London tailor 7£. 13s. 6d. 2far. for a coat; 2£. 17s. 9d. 1far. for a vest; 3£. 8s. 3d. 3far. for pantaloons; 9£. 11s. 8d. 3far. for a surtout; what was the amount of the bill? **Ans. 23£. 11s. 4d. 1far.**

OPERATION.

£.	s.	d.	far.
7	13	6	2
2	17	9	1
3	8	3	3
9	11	8	3
Ans. 23	11	4	1

Having written units of the same denomination in the same column, we find the sum of farthings in the right-hand column to be 9 farthings, equal to 2d. and 1far. We write the 1far. under the column of farthings, and carry the 2d. to the column of pence; the sum of which is 28d., equal to 2s. 4d. We write the 4d. under the column of pence, and carry the 2s. to the column of shillings; the sum of which is 51s., equal to 2£. 11s. Having written the 11s. under the column of shillings, we carry the 2£. to the column of pounds, and find the whole amount to be 23£. 11s. 4d. 1far.

The same result can be arrived at by *reducing the numbers as they are added* in their respective columns. Thus, in working the example, we can, beginning with farthings, add in this way: 3far. and 3far. are 6far., equal to 1d. 2far., and 1far. are 1d. 3far., and 2far. are 1d. 5far., equal 2d. 1far. Writing the 1far. under the column of farthings, carry the 2d. to the column of pence; add 2d. (carried) and 8d. are 10d., and 3d. are 13d., equal to 1s. 1d., and 9d. are 1s. 10d., and 6d. are 1s. 16d., equal to 2s. 4d. Writing the 4d. under the column of pence, carry the 2s. to the column of shillings; add 2s. (carried) and 11s. are 13s., and 8s. are 21s., equal to 1£. 1s., and 17s. are 1£. 18s., and 13s. are 1£. 31s., equal to 2£. 11s. Writing 11s. under the column of shillings, carry the 2£. to the column of pounds, and so find the whole amount to be, as before, 23£. 11s. 4d. 1far.

Thus the adding of compound numbers is like that of simple numbers, except in carrying; which difference holds also in subtracting, multiplying, and dividing of compound numbers.

Questions. — Art. 101. What is addition of compound numbers? How do you arrange compound numbers for addition? Why? What is the difference between addition of compound and addition of simple numbers?

RULE. — *Write all the given numbers so that units of the same denomination may stand in the same column.*

Add as in addition of simple numbers; and carry, from column to column, one for as many units as it takes of the denomination added to make a unit of the denomination next higher.

PROOF. — The proof is the same as in addition of simple numbers.

EXAMPLES FOR PRACTICE.

TROY WEIGHT.

2.

lb.	oz.	pwt.	gr.
15	11	19	22
71	10	13	17
65	9	17	14
73	11	13	13
14	8	9	9
242	4	14	3

3.

lb.	oz.	pwt.	gr.
10	10	10	10
81	11	19	23
47	7	8	19
16	9	10	14
33	10	9	21

APOTHECARIES' WEIGHT.

4.

lb	ʒ	ʒ	℈	gr.
81	11	6	1	19
75	10	7	2	13
14	9	7	1	12
37	8	1	1	11
61	11	3	2	3
272	4	3	0	18

5.

lb	ʒ	ʒ	℈	gr.
35	9	6	2	19
71	1	1	1	11
37	3	3	2	12
14	4	7	1	13
75	5	6	1	17

AVOIRDUPOIS WEIGHT.

6.

T.	cwt.	qr.	lb.	oz.	dr.
71	19	3	17	14	13
14	13	1	11	13	12
39	9	3	13	9	9
15	17	3	16	10	14
61	16	3	13	7	8
203	17	3	23	8	8

7.

T.	cwt.	qr.	lb.	oz.	dr.
14	13	2	15	15	15
13	17	3	13	11	13
46	16	3	11	13	10
14	15	2	7	6	9
11	17	3	10	15	11

QUESTIONS. — What is the rule? The proof?

CLOTH MEASURE.

8.

yd.	qr.	na.	in.
5	3	3	2
7	1	1	2
8	3	3	1
9	1	2	2
4	3	3	2
36	3	0	0

9.

E. E.	qr.	na.	in.
16	3	2	1
71	1	1	2
13	3	2	1
47	3	2	2
39	2	3	2

LONG MEASURE.

10.

deg.	m.	fur.	rd.	ft.	in.
18	19	7	15	11	1
61	47	6	39	10	11
78	32	5	14	9	9
17	59	7	36	16	10
28	56	1	30	16	1
205	8½	5	17	14½	8
	½=4			½=6	
205	9	1	17	15	2

11.

m.	fur.	rd.	yd.	ft.	in.
12	7	35	5	2	11
13	6	15	3	1	10
16	1	17	1	2	5
13	4	13	2	1	9
17	7	36	5	2	7

SURVEYORS' MEASURE.

12.

m.	fur.	ch.	p.	l.
17	5	8	3	24
16	3	7	1	21
47	7	9	3	19
19	6	6	1	16
81	7	1	0	20
133	7	4	0	0

13.

m.	fur.	ch.	p.	l.
14	7	9	3	21
37	1	0	3	16
17	7	8	3	17
61	6	5	3	16
47	1	1	0	23

SQUARE MEASURE.

14.

A.	R.	p.	ft.	in.
67	3	39	272	143
78	3	14	260	116
14	2	31	167	135
67	1	17	176	131
49	3	31	69	117
278	3	15	131¼	66
			¼=36	
278	3	15	131	102

15.

A.	R.	p.	yd.	ft.	in.
43	1	15	30	8	17
16	3	39	19	7	141
47	1	16	27	5	79
38	3	17	18	8	17
15	1	32	11	1	117

SOLID MEASURE.

16.

Tun.	ft.	in.
17	39	1371
61	17	1711
47	16	1666
71	38	1711
47	17	1617
246	11	1164

17.

Cord.	ft.	in.
14	116	1169
67	113	1711
96	127	969
19	98	1376
14	37	1414

WINE MEASURE.

18.

Tun.	hhd.	gal.	qt.	pt.
61	1	62	3	1
71	3	14	1	1
60	0	17	3	0
14	1	51	1	1
57	3	14	3	1
265	2	35	1	0

19.

Tun.	hhd.	gal.	qt.	pt.
14	3	18	3	0
81	1	60	3	1
17	3	61	3	0
61	3	57	3	1
17	1	17	1	0

BEER MEASURE.

20.

Tun.	hhd.	gal.	qt.	pt.
15	3	50	3	1
67	3	17	3	1
17	1	44	1	0
71	3	12	3	1
81	1	18	1	0
254	1	36	0	1

21.

Tun.	hhd.	gal.	qt.	pt.
67	1	51	1	0
15	3	16	3	1
44	1	45	1	1
15	2	12	2	1
67	3	35	1	0

DRY MEASURE.

22.

ch.	bu.	pk.	qt.	pt.
15	35	3	7	1
61	16	3	6	1
51	30	1	5	0
42	17	2	2	1
14	14	1	4	1
186	7	1	2	0

10*

23.

ch.	bu.	pk.	qt.	pt.
71	17	1	1	1
16	31	3	3	0
41	14	3	1	1
71	17	1	0	1
10	10	2	3	0

TIME.

24.

y.	da.	h.	m.	s.
57	300	23	59	17
47	169	15	17	38
29	364	23	42	17
18	176	16	38	47
49	317	20	52	57
203	236	10	30	56

25.

w.	da.	h.	m.	s.
15	6	23	15	17
61	5	15	27	18
71	6	21	57	58
18	5	19	39	49
87	6	19	18	57

CIRCULAR MEASURE.

26.

s.	°	′	″
11	28	56	58
10	21	51	37
8	13	39	57
8	19	38	49
7	17	47	48
11	11	55	09

27.

s.	°	′	″
6	17	17	18
7	09	19	51
8	18	57	45
4	17	16	39
7	27	38	48

NOTE. — The sum of the signs, in circular motion, must always be divided by 12, and the remainder only be written down, as in Ex. 26.

§ XII. SUBTRACTION OF COMPOUND NUMBERS.

ART. **102.** SUBTRACTION of Compound Numbers is the process of finding the difference between two compound numbers.

ENGLISH MONEY.

Ex. 1. From 87£. 9s. 6d. 3far., take 52£. 11s. 7d. 1far.

Ans. 34£. 17s. 11d. 2far.

OPERATION.

	£.	s.	d.	far.
Min.	87	9	6	3
Sub.	52	11	7	1
Rem.	34	17	11	2

Having placed the less number under the greater, farthings under farthings, pence under pence, &c., we begin with the farthings, thus: 1 far. from 3 far. leaves 2 far., which we set under the column of farthings. As we cannot

QUESTIONS. — Art. 102. What is subtraction of compound numbers? How do you arrange the numbers for subtraction?

take 7d. from 6d., we add 12d. = 1s. to the 6d., making 18d., and then subtract the 7d. from it, and set the remainder, 11d., under the column of pence. We then add 1s. = 12d. to the 11s. in the subtrahend, making 12s., to compensate for the 12d. we added to the 6d in the minuend. (Art. 30.) Again, since we cannot take 12s. from 9s., we add 20s. = 1£. to the 9s., making 29s., from which we take the 12s., and set the remainder, 17s., under the column of shillings. Having added 1£. = 20s. to the 52£., to compensate for the 20s. added to the 9s. in the minuend, we subtract the pounds as in subtraction of simple numbers, and obtain 34£. for the remainder, and as the result complete, 34£. 17s. 11d. 2far.

RULE. — *Write the less compound number under the greater, so that units of the same denomination shall stand in the same column.*

Subtract as in subtraction of simple numbers.

If any number in the subtrahend is larger than that above it, add to the upper number as many units as make one of the next higher denomination before subtracting, and carry one to the next lower number before subtracting it.

PROOF. — The proof is the same as in simple subtraction.

EXAMPLES FOR PRACTICE.

2.

£.	s.	d.	far.
78	11	5	2
41	13	3	3
36	18	1	3

3.

£.	s.	d.	far.
765	16	10	1
713	17	11	3

TROY WEIGHT.

4.

lb.	oz.	pwt.	gr.
15	3	12	14
9	11	17	21
5	3	14	17

5.

lb.	oz.	pwt.	gr.
711	1	3	17
19	3	18	19

APOTHECARIES' WEIGHT.

6.

℔	℥	ʒ	℈	gr.
15	7	1	2	15
11	9	7	1	19
3	9	2	0	16

7.

℔	℥	ʒ	℈	gr.
161	6	3	1	17
97	7	1	2	18

QUESTIONS. — What do you do when the upper number is smaller than the lower? How many do you carry to the next denomination? What is the rule for subtraction? The proof?

AVOIRDUPOIS WEIGHT.

8.

T.	cwt.	qr.	lb.	oz.	dr.
117	16	1	5	0	14
19	17	3	17	1	15
97	18	1	12	14	15

9.

T.	cwt.	qr.	lb.	oz.	dr.
11	1	0	1	1	13
9	18	3	1	13	15

CLOTH MEASURE.

10.

yd.	qr.	na.	in.
15	1	1	2
9	3	3	1
5	1	2	1

11.

E. E.	qr.	na.	in.
171	2	2	1
19	3	0	2

LONG MEASURE.

12.

deg.	m.	fur.	rd.	yd.	ft.	in.
97	3	7	31	1	1	3
19	17	1	39	1	2	7
77	55 $\frac{1}{6}$	5	31	4 $\frac{1}{2}$	1	8
	$\frac{1}{6}$ = 1		13	1	2	6
77	55	7	5	1	1	2

13.

deg.	m.	fur.	rd.	ft.	in.
18	19	1	1	3	7
9	28	7	1	16	9

SURVEYORS' MEASURE.

14.

m.	fur.	cha.	p.	l.
21	3	5	2	17
9	5	8	1	20
11	5	7	0	22

15.

m.	fur.	cha.	p.	l.
31	7	1	1	19
18	1	7	3	23

SQUARE MEASURE.

16.

A.	R.	p.	ft.	in.
116	1	13	100	113
87	3	17	200	117
28	1	35	171 $\frac{1}{4}$	140
			$\frac{1}{4}$ = 36	
28	1	35	172	32

17.

A.	R.	p.	yd.	ft.	in.
139	1	17	18	1	30
97	3	18	30	1	31

SOLID MEASURE.

18.

T.	ft.	in.
171	30	1000
98	37	1234
72	32	1494

19.

Cords.	ft.	in.
571	18	1234
199	19	1279

WINE MEASURE.

20.

f.	hhd.	gal.	qt.	pt.	gi.
171	3	8	1	1	1
99	1	19	3	1	3
72	1	51	1	1	2

21.

T.	hhd.	gal.	qt.	pt.	gi.
71	1	1	1	1	1
9	3	3	3	1	3

BEER MEASURE

22.

T.	hhd.	gal.	qt.	pt.
15	1	17	1	0
9	3	19	3	1
5	1	51	1	1

23.

T.	hhd.	gal.	qt.	pt.
79	2	2	2	0
19	3	13	3	1

DRY MEASURE.

24.

ch.	bu.	pk.	qt.	pt.
716	1	2	1	0
19	9	3	1	1
696	27	2	7	1

25.

ch.	bu.	pk.	qt.	pt.
73	13	3	0	1
19	18	1	3	1

TIME.

26.

y.	da.	h.	m.	sec.
375	15	13	17	5
199	137	15	1	39
175	243	4	15	26

27.

w.	da.	h.	m.	sec.
14	1	3	4	15
9	6	17	37	48

CIRCULAR MEASURE.

28.

S.	°	′	″
11	7	13	15
9	29	17	36
1	7	55	39

29.

S.	°	′	″
1	23	37	39
9	15	38	47
4	7	58	52

NOTE. — In Circular Measure, the minuend is sometimes less than the subtrahend, as in Ex. 29, in which case it must be increased by 12 signs

ART. **103.** To find the time between two different dates.

Ex. 1. What is the difference of time between October 16th, 1852, and August 9th, 1854? Ans. 1y. 9mo. 23da.

	FIRST OPERATION. y.	mo.	da.
Min.	1854	7	9
Sub.	1852	9	16
Rem.	1	9	23

	SECOND OPERATION.		
Min.	1854	8	9
Sub.	1852	10	16
Rem.	1	9	23

Commencing with January, the first month in the year, and counting the months and days in the later date up to August 9th, we find that 7mo. and 9 da. have elapsed; and counting the months and days in the earlier date, up to October 16th, we find that 9mo. and 16da. have elapsed. We, therefore, write the numbers for subtraction as in the first operation. The same result, however, could be obtained, as some prefer, by reckoning the *number* of the given months instead of the *number of months* that have elapsed since the beginning of the year, and writing the numbers as in the second operation; — written either way,

The earlier date being placed under the later, is subtracted, as by the preceding rule.

NOTE. — In finding the difference between two dates, and in computing interest for less than a month, 30 days are considered a month. In *legal* transactions, a month is reckoned from any day in one month to the corresponding day of the following month, if it has a corresponding day, otherwise to its end.

EXAMPLES FOR PRACTICE.

2. What is the time from March 21st, 1853, to Jan. 6th, 1857? Ans. 3y. 9m. 15da.

3. A note was given Nov. 15th, 1852, and paid April 25th, 1857; how long was it on interest?

4. John Quincy Adams was born at Braintree, Mass., July 11th, 1767, and died at Washington, D. C., Feb. 23, 1848; to what age did he live?

5. Andrew Jackson was born at Waxaw, S. C., March 15th, 1767, and died at Nashville, Tenn., June 8th, 1845; at what age did he die?

QUESTIONS. — Art. 103. From what period do you count the months and days in preparing dates for subtraction? How do you arrange the dates for subtraction? How subtract? How many days are considered a month in business transactions? What is the second method of preparing dates for subtraction?

§ XIII. MISCELLANEOUS EXERCISES IN ADDITION AND SUBTRACTION OF COMPOUND NUMBERS.

1. WHAT is the amount of the following quantities of gold: 4lb. 8oz. 13pwt. 8gr., 5lb. 11oz. 19pwt. 23gr., 8lb. 0oz. 17pwt. 15gr., and 18lb. 9oz. 14pwt. 10gr.?

Ans. 37lb. 7oz. 5pwt. 8gr.

2. An apothecary would mix 7℔ 3ʒ 2ʒ 2℈ 1gr. of rhubarb, 2℔ 10ʒ 0ʒ 1℈ 13gr. of cantharides, and 2℔ 3ʒ 7ʒ 2℈ 17gr. of opium; what is the weight of the compound?

3. Add together 17T. 11cwt. 3qr. 11lb. 12oz., 11T. 17cwt. 1qr. 19lb. 11oz., 53T. 19cwt. 1qr. 17lb. 8oz., 27T. 19cwt. 3qr. 18lb. 9oz., and 16T. 3cwt. 3qr. 0lb. 13oz.

4. A merchant owes a debt in London amounting to 7671£.; what remains due after he has paid 1728£. 17s. 9d.?

5. From 73lb. of silver there were made 26lb. 11oz. 13pwt. 14gr. of plate; what quantity remained?

6. From 71℔ 8ʒ 1ʒ 1℈ 14gr. take 7℔ 9ʒ 1ʒ 1℈ 17 gr.

7. From 28T. 13cwt. take 10T. 17cwt. 19lb. 14oz.

8. A merchant has 3 pieces of cloth; the first contains 37yd. 3qr. 3na., the second 18yd. 1qr. 3na., and the third 31yd. 1qr. 2na.; what is the whole quantity?

9. Sold 3 loads of hay; the first weighed 2T. 13cwt. 1qr. 17lb., the second 3T. 17lb., and the third 1T. 3qr. 11lb.; what did they all weigh?

10. What is the sum of the following distances: 16m. 7fur. 18rd. 14ft. 11in., 19m. 1fur. 13rd. 16ft. 9in., 97m. 3fur. 27rd. 13ft. 3in., and 47m. 5fur. 37rd. 13ft. 10in.?

11. From 76yd. take 18yd. 3qr. 2na. Ans. 57yd. 0qr. 2na.

12. From 20m. take 3m. 4fur. 18rd. 13ft. 8in.

13. From 144A. 3R. take 18A. 1R. 17p. 200ft. 100in.

14. From 18 cords take 3 cords 100ft. 1000in.

15. A gentleman has three farms; the first contains 169A. 3R. 15p. 227ft., the second 187A. 1R. 15p. 165ft., and the third 217A. 2R. 28p. 165ft.; what is the whole quantity?

16. There are 3 piles of wood; the first contains 18 cords 116ft. 1000in., the second 17 cords 111ft. 1600in., and the third 21 cords 109ft. 1716in.; how much in all?

17. From 17T. take 5T. 18ft. 765 in.

18. From 169gal. take 76gal. 3qt. 1pt.

19. From 17ch. 18bu. take 5ch. 20bu. 1pk. 7qt.

20. From 83y. take 47y. 10mo. 27d. 18h. 50m. 14s.

21. From 11S. 15° 36′ 15″ take 5S. 18° 50′ 18″.

22. John Thomson has 4 casks of molasses; the first contains 167gal. 3qt. 1pt., the second 186gal. 1qt. 1pt., the third 108gal. 2qt. 1pt., and the fourth 123gal. 3qt. 0pt.; how much is the whole quantity?

23. Add together 17bu. 1pk. 7qt. 1pt., 18bu. 3pk. 2qt., 19bu. 1pk. 3qt. 1pt., and 51bu. 3pk. 0qt. 1pt.

24. James is 13y. 4mo. 13d. old, Samuel is 12y. 11mo. 23d., and Daniel is 18y. 9mo. 29d.; what is the sum of their united ages?

25. Add together 18y. 345d. 13h. 37m. 15s., 87y. 169d. 12h. 16m. 28s., 316y. 144d. 20h. 53m. 18s., and 13y. 360d. 21h. 57m. 15s.

26. A carpenter sent two of his apprentices to ascertain the length of a certain fence. The first stated it was 17rd. 16ft. 11in., the second said it was 18rd. 5in. The carpenter, finding a discrepancy in their statements, and fearing they might both be wrong, ascertained the true length himself, which was 17rd. 5yd. 1ft. 11in.; how much did each differ from the other?

27. From a mass of silver weighing 106lb., a goldsmith made 36 spoons, weighing 5lb. 11oz. 12pwt. 15gr.; a tankard, 3lb. 0oz. 13pwt. 14gr.; a vase, 7lb. 11oz. 14pwt. 23gr.; how *much* unwrought silver remains?

28. From a piece of cloth, containing 17yd. 3qr., there were taken two garments, the first measuring 3yd. 3qr. 2na., the second 4yd. 1qr. 3na.; how much remained?

29. Venus is 3S. 18° 45′ 15″ east of the Sun, Mars is 7S. 15° 36′ 18″ east of Venus, and Jupiter is 5S. 21° 38′ 27″ east of Mars; how far is Jupiter east of the Sun? Ans. 4S. 26°.

30. The longitude of a certain star is 3S. 18° 14′ 35″, and the longitude of Jupiter is 11S. 25° 30′ 50″; how far will Jupiter have to move in his orbit to be in the same longitude with the star?

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§ XIV. MULTIPLICATION OF COMPOUND NUMBERS.

ART. **104.** MULTIPLICATION of Compound Numbers is the process of taking a compound number any proposed number of times.

ART. **105.** To multiply when the multiplier is not more than 12.

Ex. 1. If an acre of land cost 14£. 5s. 8d. 2far., what will 9 acres cost? Ans. 128£. 11s. 4d. 2far.

OPERATION.

	£.	s.	d.	far.
Multiplicand	14	5	8	2
Multiplier				9
Product	128	11	4	2

We write the multiplier under the lowest denomination of the multiplicand, and then say 9 times 2far. are 18far., equal to 4d. and 2far. We set down the 2far. under the number multiplied, reserving the 4d. to be added to the next product. We then say 9 times 8d. are 72d., and the 4d. make 76d., equal to 6s. and 4d., and set the 4d. under the column of pence, reserving the 6s. to be added to the next product. Then, 9 times 5s. are 45s., and 6s. make 51s., equal to 2£. and 11s. We place the 11s. under the column of shillings, reserving the 2£. to be added to the next product. Again, 9 times 14£. are 126£., and 2£. make 128£. This, placed under the column of pounds, gives us 128£. 11s. 4d. 2far. for the answer.

QUESTIONS. — Art. 104. What is multiplication of compound numbers? — Art. 105. Explain the operation. By what do you divide the product of each denomination? What do you do with the quotient and remainders thus obtained?

RULE. — *Multiply each denomination of the compound number as in multiplication of simple numbers, and carry as in addition of compound numbers.*

NOTE. — Going a second time carefully over the work is a good way of testing its accuracy On learning Division of Compound Numbers, the pupil will find that rule a better method of proving multiplication of compound numbers.

EXAMPLES FOR PRACTICE.

2.

£.	s.	d.
5	6	8
		2
10	13	4

3.

£.	s.	d.
19	11	7
		3
58	14	9

4.

£.	s.	d.
25	17	11
		5
129	9	7

5.

£.	s.	d.
18	15	$8\frac{3}{4}$
		6
112	14	$4\frac{1}{2}$

6.

cwt.	qr.	lb.	oz.
18	3	17	10
			6
113	2	5	12

7.

Ton.	cwt.	qr.	lb.
14	15	3	12
			7
103	11	0	9

8.

cwt.	qr.	lb.	oz.
19	1	8	15
			8
154	2	21	8

9.

lb.	oz.	dr.
15	14	13
		9
143	5	5

10.

m.	fur.	rd.	ft.
97	7	14	13
			6
587	4	8	12

11.

deg.	m.	fur.	rd.
18	12	6	18
			8
145	33	2	$10\frac{2}{3}$

12.

rd.	yd.	ft.	in.
23	3	2	9
			9
213	2	0	9

13.

fur.	rd.	ft.	in.
9	31	16	11
			10
98	0	4	2

NOTE. — The answers to the following questions are found in the corresponding questions in Division of Compound Numbers, p. 128.

14. What cost 7 yards of cloth at 18s. 9d. per yard?

15. If a man travel 12m. 3fur. 29rd. in one day, how far will he travel in 9 days?

16. If 1 acre produce 2 tons 13cwt. 19lb. of hay, what will 8 acres produce?

QUESTIONS. — What is the rule? How may the work be tested?

17. If a family consume 49gal. 3qt. 1pt. of molasses in one month, what quantity will be sufficient for one year?

18. John Smith has 12 silver spoons, each weighing 3oz. 17pwt. 14gr.; what is the weight of all?

19. Samuel Johnson bought 7 loads of timber, each measuring 7 tons 37ft.; what was the whole quantity?

20. If the moon move in her orbit 13° 11′ 35″ in 1 day, how far will she move in 10 days?

21. If 1 dollar will purchase 2℔ 8℥ 7℥ 1℈ 10gr. of ipecacuanha, what quantity would 9 dollars buy?

22. If 1 dollar will buy 2A. 3R. 15p. 30yd. 8ft. 100in. of wild land, what quantity may be purchased for 12 dollars?

23. Joseph Doe will cut 2 cords 97ft. of wood in 1 day; how much will he cut in 9 days?

24. If 1 acre of land produce 3ch. 6bu. 2pk. 7qt. 1pt. of corn, what will 8 acres produce?

ART. **106.** When the multiplier is a composite number, and none of its factors exceed 12.

Ex. 1. What cost 24 yards of broadcloth at 2£. 7s. 11d. per yard? Ans. 57£. 10s. 0d.

OPERATION.

£.	s.	d.	
2	7	11	= price of 1 yard.
		4	
9	11	8	= price of 4 yards.
		6	
57	10	0	= price of 24 yards.

We find the number 24 equal to the product of 4 and 6; we therefore multiply the price first by 4, and then that product by 6, and the last product is the answer.

Ex. 2. What cost 360 tons of iron at 17£. 16s. 1d. per ton? Ans. 6409£. 10s. 0d.

OPERATION.

£.	s.	d.	
17	16	1	= price of 1 ton.
		6	
106	16	6	= price of 6 tons.
		6	
640	19	0	= price of 36 tons.
		10	
6409	10	0	= price of 360 tons.

We find the factors of 360 to be 6, 6, and 10. We first multiply by 6, and then that product by 6, and then again the last product by 10.

RULE. — *Multiply by the factors of the composite number in succession.*

EXAMPLES FOR PRACTICE.

3. If a man travel 3m. 7fur. 18rd. in one day, how far would he travel in 30 days?

4. If a load of hay weigh 2 tons 7cwt. 3qr. 18lb., what would be the weight of 84 similar loads?

5. When it requires 7yd. 3qr. 2na. of silk to make a lady's dress, what quantity would be sufficient to make 72 similar dresses?

6. A tailor has an order from the navy agent to make 132 garments for seamen; how much cloth will it take, supposing each garment to require 3yd. 2qr. 1na.?

ART. 107. When the multiplier is not a composite number, and exceeds 12, or, if a composite number, and any of its factors exceed 12.

Ex. 1. What cost 379cwt. of iron at 3£. 16s. 8d. per cwt.?
Ans. 1452£. 16s. 8d.

OPERATION.

£.	s.	d.	
3	16	8	× 9 units.
	10		
38	6	8	× 7 tens.
	10		
383	6	8	
		3	hundreds.
1150	0	0	cost of 300cwt.
268	6	8	cost of 70cwt.
34	10	0	cost of 9cwt.
1452	16	8	cost of 379cwt.

Since 379 is not a composite number, we cannot resolve it into factors; but we may separate it into parts, and find the value of each part separately; thus, 379 = 300 + 70 + 9. In the operation, we first multiply by 10, and then this product by 10, to get the cost of 100cwt. To find the cost of 300cwt., we multiply the last product by 3; and to find the cost of 70cwt., we multiply the cost of 10cwt. by 7: and then, to find the cost of 9cwt., we multiply the cost of 1cwt. by 9. Adding the several products, we obtain 1452£. 16s. 8d. for the answer.

RULE. — *Having resolved the multiplier into any convenient parts, as of units, tens, &c., multiply by these several parts, adding together the products thus obtained for the required result.*

QUESTIONS. — Art. 106. What is the rule for multiplying by a composite number? Give the reason for the rule. — Art. 107. How do you find the cost of 300cwt. in the example? Of 70cwt.? Of 9cwt.? What is the rule *when* the multiplier is large, and is not a composite number?

EXAMPLES FOR PRACTICE.

2. If 1 dollar will buy 17lb. 10oz. 13dr. of beef, how much may be bought for 62 dollars?

3. What cost 97 tons of lead at 2£. 17s. 9½d. per ton?

4. If a man travel 17m. 3fur. 19rd. 3yd. 2ft. 7in. in one day, how far would he travel in 38 days?

5. If 1 acre will produce 27bu. 3pk. 6qt. 1pt. of corn, what will 98 acres produce?

6. If it require 7yd. 3qr. 2na. to make 1 cloak, what quantity would it require to make 347 cloaks?

7. One ton of iron will buy 13A. 3R. 14p. 18yd. 7ft. 76in. of land; how many acres will 19 tons buy?

8. If 1 ton of copper ore will purchase 17T. 14cwt. 3qr. 18lb. 14oz. of iron ore, how much can be purchased for 451 tons?

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§ XV. DIVISION OF COMPOUND NUMBERS.

ART. **108.** DIVISION of Compound Numbers is the process of dividing compound numbers into any proposed number of equal parts.

ART. **109.** To divide when the divisor does not exceed 12.

EX. 1. If 9 acres of land cost 128£. 11s. 4d. 2far., what is the value of 1 acre? Ans. 14£. 5s. 8d. 2far.

OPERATION.

	£.	s.	d.	far.
9)1 2 8	1 1	4	2
1 4	5	8	2

Having divided the 128£. by 9, we find the quotient to be 14£. and 2£. remaining. We place the quotient 14£. under the 128£., and to the remainder 2£., equal to 40s., we add the 11s. in the question, and divide the amount, 51s., by 9. We write the quotient 5s. under the 11s., and to the remainder 6s., equal to 72d., we add the 4d., making 76d., which we divide by 9, and write the quotient 8d. under the 4d. To the remainder 4d., equal to 16far., we add

QUESTIONS. — Art. 108. What is division of compound numbers? — Art. 109. Where do you begin to divide? Why? When there is a remainder after dividing any one denomination, what must be done with it?

the 2far., and divide the amount. 18far., by 9, and obtain 2far. for a quotient, which we place under the 2far. in the dividend. Thus we find the answer to be 14£. 5s. 8d. 2far.

RULE. — *Divide as in division of simple numbers, each denomination in its order, beginning with the highest.*

If there be a remainder, reduce it to the next lower denomination, adding in the number already of this denomination, if any, and divide as before.

PROOF. — The same as in simple numbers.

NOTE. — When the divisor and dividend are both compound numbers, they must be reduced to the same denomination, and the division then is that of simple numbers.

2.

£.	s.	d.
2)10	13	4
5	6	8

3.

£.	s.	d.
3)58	14	9
19	11	7

4.

£.	s.	d.
5)129	9	7
25	17	11

5.

£.	s.	d.	far.
6)112	14	4	2
18	15	8	3

6.

cwt.	qr.	lb.	oz.
6)113	2	5	12
18	3	17	10

7.

ton.	cwt.	qr.	lb.
7)103	11	0	9
14	15	3	12

8.

cwt.	qr.	lb.	oz.
8)154	2	21	8
19	1	8	15

9.

lb.	oz.	dr.
9)143	5	5
15	14	13

10.

m.	fur.	rd.	ft.
6)587	4	8	12

11.

deg.	m.	fur.	rd.
8)145	33	2	$10\frac{3}{4}$

12.

rd.	yd.	ft.	in.
9)213	2	0	9

13.

fur.	rd.	ft.	in.
10)98	0	4	2

NOTE. — The answers to the following questions are found in the corresponding numbers in Multiplication of Compound Numbers.

14. What costs 1 yard of cloth, when 7yd. can be bought for 6£. 11s. 3d.?

15. If a man, in 9 days, travel 112m. 1fur. 21rd., how far will he travel in 1 day?

16. If 8 acres produce 21T. 5cwt. 2qr. 2lb. of hay, what will 1 acre produce?

QUESTION. — What is the rule for division of compound numbers?

17. If a family consume in 1 year 598 gal. 2qt. of molasses, how much will be necessary for 1 month?

18. John Smith has 12 silver spoons, weighing 3lb. 10oz. 11pwt.; what is the weight of each spoon?

19. Samuel Johnson bought 7 loads of timber, measuring 55T. 19ft.; what was the quantity in each load?

20. If the moon, in 10 days, move in her orbit 4S. 11° 55′ 50″, how far does she move in 1 day?

21. If \$9 will buy 24℔ 8℥ 3℥ 1℈ 10gr. of ipecacuanha, how large a quantity will \$1 purchase?

22. When \$12 will buy 34A. 0R. 32p. 8yd. 5ft. 48in. of wild land, how much will \$1 buy?

23. Joseph Doe will cut 24 cords 105 feet of wood in 9 days; how much will he cut in 1 day?

24. When 8 acres of land produce 25ch. 17bu. 3pk. 4qt. of grain, what will 1 acre produce?

ART. **110.** When the divisor is a composite number, and none of its factors exceed 12.

Ex. 1. When 24 yards of broadcloth are sold for 57£. 10s. 0d., what is the price of 1 yard? Ans. 2£. 7s. 11d.

OPERATION.

£.	s.	d.	
6)57	10	0	= price of 24 yards.
4)9	11	8	= price of 4 yards.
2	7	11	= price of 1 yard.

We find the component parts, or factors, of 24, are 6 and 4. We therefore divide the price by one of these numbers, and the quotient by the other.

RULE. — *Divide by the factors of the composite number in succession.*

EXAMPLES FOR PRACTICE.

2. If 360 tons of iron cost 6409£. 10s. 0d., what is the cost of 1 ton?

3. If a man travel 117m. 7fur. 20rd. in 30 days, how far will he travel in 1 day?

4. If 84 loads of hay weigh 201 tons 6cwt. 0qr. 12lb., what will 1 load weigh?

5. When 72 ladies require 567yd. 0qr. 0na. for their dresses, how many yards will be necessary for one lady?

QUESTIONS. — Art. 110. How does it appear that dividing by 6 in Ex. 1 gives the price of 4 yards? How do you divide by a composite number?

6. When 132 sailors require 470yd. 1qr. of cloth to make their garments, how many yards will be necessary for 1 sailor?

ART. 111. When the divisor is not a composite number, and exceeds 12, or, if a composite number, and any of its factors exceed 12, *the whole operation can be written down*, as in the following example:

Ex. 1. If 23cwt. of iron cost 171£. 1s. 3d., what cost 1cwt.? Ans. 7£. 8s. 9d.

OPERATION.

£. s. d.

23)171 1 3(7£.
161

10
20

23)201(8s.
184

17
12

23)207(9d.
207

We divide the pounds by 23, and obtain 7 for the quotient, and 10£. remaining, which we reduce to shillings, and add the 1s., and again divide by 23, and obtain 8s. for the quotient. The remainder, 17s., we reduce to pence, and add the 3d., and again divide by 23, and obtain 9d. for the quotient. Thus, the method of operation is the same as by the general rule (Art. 109), excepting more of the work is written down; and, by uniting the several quotients, we find the answer to be 7£. 8s. 9d

2. If $62 will buy 1095lb. 14oz. 6dr. of beef, how much may be obtained for $1?

3. Paid 280£. 5s. 9½d. for 97 tons of lead; what did it cost per ton?

4. If a man travel 662m. 4fur. 28rd. 3yd. 2ft. 2in. in 38 days, how far will he travel in 1 day?

5. When 98 acres produce 2739 bu. 1pk. 5qt. of grain, what will 1 acre produce?

6. A tailor made 347 garments from 2732yd. 2qr. 2na. of cloth; what quantity did it take to make 1 garment?

7. When 19 tons of iron will purchase 262A. 3R. 37p. 25yd. 1ft. 40in. of land, how much may be obtained for 1 ton?

8. If 451 tons of copper ore will purchase 8003T. 17cwt. 1qr. 12lb. 10oz. of iron ore, how much will 1 ton purchase?

QUESTION. — Art. 111. When the divisor is large, and not a composite number, *how* is the division performed?

§ XVI. MISCELLANEOUS EXAMPLES IN MULTIPLICATION AND DIVISION OF COMPOUND NUMBERS.

1. BOUGHT 30 boxes of sugar, each containing 8cwt. 3qr. 20lb., but having lost 68cwt. 2qr. 0lb., I sold the remainder for 1£. 17s. 6d. per cwt.; what sum did I receive?

2. A company of 144 persons purchased a tract of land containing 11067A. 1R. 8p. John Smith, who was one of the company and owned an equal share with the others, sold his part of the land for 1s. 9½d. per square rod; what sum did he receive?

3. The exact distance from Boston to the mouth of the Columbia River is 2644m. 3fur. 12rd. A man, starting from Boston, travelled 100 days, going 18m. 7fur. 32rd. each day; required his distance from the mouth of the Columbia at the end of that time.

4. James Bent was born July 4, 1798, at 3h. 17m. A. M.; how long had he lived Sept. 9, 1807, at 11h. 19m. P. M., reckoning 365 days for each year, excepting the leap year 1804, which has 366 days?

5. The distance from Vera Cruz, in a straight line, to the city of Mexico, is 121m. 5fur. If a man set out from Vera Cruz to travel this distance, on the first day of January, 1848, which was Saturday, and travelled 3124rd. per day until the eleventh day of January, omitting, however, as in duty bound, to travel on the Lord's day, how far would he be from the city of Mexico on the morning of that day?

6. Bought 16 casks of potash, each containing 7cwt. 3qr. 18lb., at 5 cents per pound. I disposed of 9 casks at 6 cents per pound, and sold the remainder at 7 cents per pound; what did I gain?

7. A merchant purchased in London 17 bales of cloth for 17£. 18s. 10d. per bale. He disposed of the cloth at Havana for sugar at 1£. 17s. 6d. per cwt. Now, if he purchased 144cwt. of sugar, what balance did he receive?

8. A and B commenced travelling, the same way, round an island 50 miles in circumference. A travels 17m. 4fur. 30rd. a day, and B travels 12m. 3fur. 20rd. a day; required how far they are apart at the end of 10 days.

9. Bought 760 barrels of flour at $5.75 per barrel, which I paid for in iron at 2 cents per pound. The purchaser afterwards sold one half of the iron to an axe manufacturer; what quantity did he sell?

10. Bought 17 house-lots, each containing 44 perches, 200 square feet. From this purchase I sold 2A. 2R. 240ft., and the remaining quantity I disposed of at 1s. 2½d. per square foot; what amount did I receive for the last sale?

11. J. Spofford's farm is 100 rods square. From this he sold H. Spaulding a fine house-lot and garden, containing 5A. 3R. 17p., and to D. Fitts a farm 50rd. square, and to R. Thornton a farm containing 3000 square rods; what is the value of the remainder, at $1.75 per square rod?

12. Bought 78A. 3R. 30p. of land for $7000, and, having sold 10 house-lots, each 30rd. square, for $8.50 per square rod, I dispose of the remainder for 2 cents per square foot. How much do I gain by my bargain?

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Also, in the *Connecticut State Normal School*, NEW BRITAIN; *French's Classical and Scientific Boarding and Day School for Boys, Russell's Collegiate and Commercial Institute, York Square Seminary*, NEW HAVEN; *Harris' English Classical and Commercial Academy, Hartford Female Seminary, American Asylum*, HARTFORD; *State Reform School*, WEST MERIDEN; *Connecticut Literary Institution*, SUFFIELD; *Bacon Academy*, COLCHESTER; *Academy*, PLAINFIELD; *Collegiate Institute*, MIDDLETOWN; and other similar institutions; and in *Select*, or *Public Schools*, in the following, among other places, in CONNECTICUT:

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NEW HAVEN,	BRISTOL,	EAST WINDSOR,	BIRMINGHAM,
NORWICH,	ENFIELD,	PLAINFIELD,	WATERBURY,
NEW LONDON,	SUFFIELD,	MILFORD,	NORWALK,
BRIDGEPORT,	WOODSTOCK,	MERIDEN,	BROOKLYN,
STAMFORD,	STRATFORD,	WESTPORT,	COLCHESTER,
EAST HARTFORD,	FARMINGTON,	GLASTENBURY,	MANCHESTER,
NEW BRITAIN,	SOUTHINGTON,	WETHERSFIELD,	HADDAM,
EAST HADDAM,	SAYBROOK,	GROTON,	LYME,
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The BOARD OF EDUCATION of NEW YORK CITY, in 1859, adopted this *entire* ARITHMETICAL SERIES for use in the *Public Schools* in that city, where the books are now much used.

The SERIES, in whole or part, is used in the *New York Normal School, Dolbear's Commercial Academy*, and in other important Seminaries in NEW YORK CITY; *Packer Collegiate Institute, Prof. Alonzo Gray's, Prof. Greenleaf's*, and other distinguished Institutions in BROOKLYN; *Williamsburg Institute, Amenia Seminary, Binghampton Academy, Brockport Collegiate Institute, Clinton Liberal Institute, Starkey Seminary, Kinderhook Academy, North Granville Female Seminary, Union Hall Academy*, &c.; and is adopted for *Public Schools* in important Cities in NEW YORK STATE.

The CONTROLLERS of the PUBLIC SCHOOLS of the CITY and COUNTY of PHILADELPHIA, December, 1859, adopted the COMMON SCHOOL and NATIONAL ARITHMETICS for the Public Schools of that District; and the *series* is used in the schools generally in most of the CITIES and LARGER BOROUGHS OF PENNSYLVANIA.

The following *Seminaries* of a high grade, among others, in the state of PENNSYLVANIA, are using, in part or whole, GREENLEAF'S MATHEMATICAL SERIES:

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Spring Garden Ins. for Girls, "
Supplie's Ins. for Y. Ladies, "
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Spring Garden Ins. for Boys, "
Central Institute, "
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Osler Avenue School, "
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Summer-street Institute, "
Kendall's Classical and English School, "
Friends' Cent. High School, "
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Short's Eng. and Class. Sch., "
Spring Garden Academy, "
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Young's Ins. for Y. Ladies, "
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Langton's Academy, "
Ashton's Institute for Y. L., "
Hemans Institute, "
Reynold's Col. & Com. Acad., "
Miss Robb's Inst. for Y. L. "
Alexander's Eng. and Class. Academy, "
University Fem. Inst., LEWISBURG.
Boarding School, POTTSTOWN.
Union School, INDIANA.
Academy, SHADE GAP.
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Public High School, "
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Wayne Co. Normal Sch., BROMPTON.
Penn. Female College, HARRISBURG.
Harrisburg Academy, "
Public High Schools, "
Reading Institute, READING.
Public High School, "
Pottsville Academy, POTTSVILLE.
Public High School, "
State Normal School, MILLERSVILLE.
Oakland Female Ins., NORRISTOWN.
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York Co. Academy, for Y. L., "
Public High School, "
Penn. Commercial College, "
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Public High School, "
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Germantown Academy, GERMANTOWN.
Price Street Academy, "
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Western University, "
English and Classical School, "
Oakland Seminary, "
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Classical Institute, MEDIA.
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Academy, LEWISTOWN.
Graded Union School, SCRANTON.
Academy, ALLENTOWN.
Public High School, "
Public High School, SELINGSGROVE.
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College,	NEW WILMINGTON.	*Normal Institute,*	MARKLEVILLE.
Academy,	WILLIAMSBURG.	*Mt. Dempsy Academy,*	LANDISBURG.
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Young Ladies Sem.,	ERCILDOWN.	*Public High School,*	"
Seminary,	BERRYSBURG.	*Public High School,*	CHAMBERSBURG.
Cumberland Valley Institute,	MECHANICSBURG.	*Marion Select Academy,*	PLAINFIELD.
		Public High School,	HUNTINGDON.
Irving Female College,	"	*Academy,*	MACALLISTERVILLE.
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Cedar Hill Seminary,	MOUNT JOY.	*Graded School,*	GETTYSBURG.
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College,	WASHINGTON.	*Academy,*	HOLLIDAYSBURG.
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The SERIES, in whole, or in part, has been officially adopted by directors for many *Districts* in the following COUNTIES, in PENNSYLVANIA:

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Baltimore Commercial College,	*Principal,*	PROF. E. K. LOZIER.
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Union Academy,	"	MESSRS. WAUGH & MAGERS.
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Calvert Hall,	"	THE CHRISTIAN BROTHERS.

Also, in *St. John's College,* ANNAPOLIS; *Mrs. Lincoln Phelps' Female Institute,* ELLICOTT'S MILLS; *College, Female Seminary, Sawtell and French's Seminary, Academy of Visitation,* FREDERICK CITY; *Academy,* EMMETTSBURG; *Academy, High School,* WESTMINSTER; *Academy, Female Seminary, High School,* FROSTBURG, and other institutions in MARYLAND.

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GREENLEAF'S

NEW PRIMARY ARITHMETIC,

DESIGNED FOR PRIMARY CLASSES.

By BENJAMIN GREENLEAF, A. M.,

AUTHOR OF A MATHEMATICAL SERIES.

These First Lessons in numbers surpasses all others: —

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5. *In being adapted to precede any good Intellectual Arithmetic, but especially the next book in Greenleaf's Series.*

Recommendations.

From Miss M. A. Neal, *Teacher of one of the Lowell Primary Schools.*

Lowell, Dec. 3, 1859.

Having used Greenleaf's New Primary Arithmetic in my third class for the last three months, it gives me pleasure to say that I regard it as an excellent introduction to Colburn's First Lessons in Arithmetic, now in use.

MARTHA A. NEAL.

Lowell, July, 1858.

Having used Greenleaf's New Primary Arithmetic in my school for the last six months, I can heartily recommend it to all as the best in use for beginners.

MARY A. BEARD,
Teacher of Primary School No. 36.

Primary School No. 41, Lowell, July 20, 1858.

I have used Greenleaf's New Primary Arithmetic, for the last six months, in the third class in my school, and think it decidedly the best Arithmetic for beginners I have ever examined.

M. R. KITTREDGE.

GREENLEAF'S

INTELLECTUAL ARITHMETIC,

DESIGNED FOR COMMON SCHOOLS AND ACADEMIES.

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THIS advanced intellectual course has been expressly prepared to meet the highest requirements of the *present* improved standard of mental culture, and is regarded, by eminent Teachers who have used the work, to be far superior to any other book of the kind: —

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4. *In giving prominence to* UNITED STATES MONEY, *while in Colburn's First Lessons there are numerous exercises in "pounds, shillings, and pence."*

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6. *It contains exercises for drilling the learner in performing mental operations on* LARGE NUMBERS *and* DECIMAL FRACTIONS; *an important feature peculiar to this work.*

7. *It includes* ALL *the* PRINCIPLES *and* COMBINATIONS *common to books of the kind, and* OTHERS CARRYING ANALYSIS FURTHER, *so that a pupil, who may have thoroughly mastered Colburn's First Lessons, might fail to perform all the exercises, even in common fractions, in Greenleaf's Intellectual.*

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GREENLEAF'S

COMMON SCHOOL ARITHMETIC,

Designed for Common Schools and Academies.

BY BENJAMIN GREENLEAF, A. M.,

AUTHOR OF A MATHEMATICAL SERIES.

This work is on the inductive plan, and combines all the excellences of both the analytic and synthetic methods of instruction. It has been expressly prepared as "*the book for the million*," and, from the admirable results produced by its use, has been pronounced, by prominent educators, in all parts of the country, to be superior to all other works of the kind:

1. *In its* ARRANGEMENT, *being strictly inductive and logical.*

2. *In its* DEFINITIONS, RULES, EXPLANATIONS *and* DEMONSTRATIONS, *being clear, accurate and concise.*

3. *In the* NUMBER, VARIETY *and* PRACTICAL CHARACTER, *of the* EXAMPLES, *calculated to discipline the mind of the learner, and at the same time prepare for the business operations of life.*

4. *In the* ARTICLES *on* MONEY, WEIGHTS, MEASURES, INTEREST, CUSTOM HOUSE BUSINESS, &c., *being strictly conformable to present usage, and accompanied with suitable notes and explanations.*

5. *In the prominence given to* ANALYSIS, *or "the common-sense rule," in all parts of the work, — the rules for the most part being deduced from the analytic solution of one or more problems.*

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7. *In* AVOIDING OLD RULES AND DISTINCTIONS, *which modern improvements have rendered unnecessary.*

8. *In being a* COMPLETE SYSTEM OF WRITTEN ARITHMETIC *for Common Schools, containing sufficient, in itself,* to prepare the learner for all ordinary *business.*

From PROF. N. H. MAGUIRE, *Principal of Central High School, Philadelphia.*

I have examined Greenleaf's Arithmetics, Algebra and Geometry, in conjunction with Prof. M'CLUNE (Professor of Higher Mathematics in this institution); and we have heartily concurred in the opinion that they are the best works of the kind published. NICHOLAS H. MAGUIRE.

PHILADELPHIA, Jan. 25, 1859.

From MARTHA H. RODGERS, *Teacher of Mathematics in the Girls' High School.*

I have used Greenleaf's Arithmetic for some years, and have found its examples, from their *practical* character, *admirably* adapted to enable pupils to *apply* the knowledge of the rules taught, and, from the prominence given to the *analytical* mode of solution, well calculated to improve and strengthen the reasoning powers.

PHILADELPHIA, Oct. 11, 1859. MARTHA H. RODGERS.

I fully concur in the views expressed by Miss Rodgers, as above written.

P. A. CREGAR, *Principal of Girls' High School, Philadelphia.*

From WILLIAM ROBERTS, *late President of Pennsylvania State Teachers' Association.*

I have long been sensible of the merits of Greenleaf's excellent system of Arithmetic, and cheerfully join my fellow-laborers in the recommendation of the Common School and National Arithmetics by that author.

The clearness and conciseness of the rules and definitions, the judicious and varied examples for practice, entitle both works to a high rank in our school classics.

I sincerely hope they may be introduced into our public schools and other institutions of learning throughout the country.

WILLIAM ROBERTS, *Principal of Ringgold Gram. School.*

PHILADELPHIA, Oct. 31, 1859.

PHILADELPHIA, Oct. 24th, 1859.

During seven years I have had "Greenleaf's National Arithmetic" in constant use, and have found it to be a very serviceable and superior work. I have also examined the other mathematical works included in the series issued by Benjamin Greenleaf, and am of the opinion that, in conciseness of rules, accuracy of definitions, and scope of examples, it is unsurpassed by any other series with which I am conversant. ALEX. H. LAIDLAW, A. M.,

Principal of the Monroe Boys' Grammar School.

The undersigned fully concur in the favorable opinions expressed above of Greenleaf's Mathematical series.

JAMES H. ELDRIDGE, A. M.,
Principal of Hancock Boys' Grammar School.
EDWARD GIDEON, A. M.,
Principal of Penn. Boys' Grammar School.
WILLIAM STEPHENS, A. M.,
Principal of J. Q. Adams Grammar School.
PHILIP CRESSMAN, A. M.,
Principal of S. W. Boys' Grammar School.
WILLIAM STIRLING, A. M.,
Principal of Locust Street School.

PHILADELPHIA, October 12th, 1859.

To the request for an expression of my opinion of "Greenleaf's Mathematical Series," it gives me pleasure to say,—I have used these books for several years, *and regard them as very superior works*; forming, in fact, the most complete system *of which I* have any knowledge. GEO. W. FETTER,

Principal of Mount Vernon Male Gram. School.

From the PRESIDENT *of Philadelphia Teachers' Association.*

PHILADELPHIA, Nov. 3, 1859.

I have examined Greenleaf's Arithmetics, and have no hesitation in expressing my decided approbation of the series. The gradual manner in which the pupil is led on, from the first principles to the more difficult parts, by the numerous practical examples, clear elucidations, and the judicious arrangement of the same, renders it a valuable contribution to our means of instruction, and, in my opinion, one of the best productions on the subject extant.

JNO. SICKEL, *Principal of Jefferson Boys' Gram. School.*

From the PRINCIPAL *of North East Boys' Grammar School, Philadelphia.*

PHILADELPHIA, Oct. 14, 1859.

The best examination to which any text-book can be submitted, is the ordeal of daily use in the school-room. That test I thoroughly applied to Greenleaf's Arithmetics, when principal of another school; and though, both before and since, I have been compelled to use many other works, yet *none, in my estimation, surpass the above named in disciplining the mind, as well as in giving a practical knowledge of numbers.* I had supposed them almost incapable of improvements; but, on examining the copies of the revised editions, I find many valuable additions. With such works as Greenleaf's in the hands of one's pupils, Arithmetic becomes attractive, and teaching it a pleasure.

RICHARD S. JAMES, A.M., *Principal.*

From TEACHERS *of Public Schools in Easton, Pa.*

EASTON, PA., Feb. 13, 1860.

BENJAMIN GREENLEAF, ESQ.:

Some of the undersigned have used your series of Arithmetics for nearly eight years, and others for a less term, in the public schools of this borough, but we *all* agree in saying that we regard them as superior to any others with which we are acquainted. The prominent features of the works are clearness and precision of the rules and definitions, the great number and variety of the practical examples, and their adaptation to all classes of pupils. The series shows plainly the hand of experience. The only true test of a school-book is the school-room, and these books have nobly borne that.

B. F. STEM, *Teacher of Classical and Mathematical High School.*
J. RUSSELL DUTTON, *Teacher of Female High School.*
H. P. MARSTON, *Teacher in Male Grammar School.*
SETH I. THORP, *Teacher of Bush Ward Grammar School.*
M. L. BIDLEMAN, *Teacher of Female Grammar School.*
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GILBERT TERWILLIGER, *Teacher of Grammar School No.* 2.
F. C. TOLLES, *Teacher of Secondary School.*
J. W. WEAVER, *Teacher of Secondary School.*

YORK, Nov. 25, 1859.

I have used Greenleaf's Arithmetics for several years as text-books in my Commercial School, and regard them as the very best books of the kind extant. Their definitions are decidedly clearer than those of any other work I have examined, the analysis more rigid, and the rules are remarkably brief and accurate. The series has been adopted for the public schools of the borough, and is considered to be an indispensable acquisition.

T. KIRK WHITE,
Principal of Penn. Commercial College, York, Pa.

From the TEACHERS OF THE PUBLIC SCHOOLS *of Reading, Pa.*

READING, PA., JAN. 16, 1860.

For a period of about ten years Greenleaf's Arithmetics have been standard class-books in the public schools of this city, and during that time the revisions and improvements of these works have kept them in public favor, and superseded the necessity of adopting other books of the kind. In consequence of their progressive and logical arrangement, admirable selection of examples, simplicity of rules, and clearness of definitions, we do not hesitate to pronounce the "Common School" and "National" unqualifiedly the best text-books on Arithmetic with which we are acquainted.

J. F. VALENTINE, *Principal of High School.*
LEVI A. STEWART, *1st Male Assistant in High School.*
GEO. F. WELLS, *2d Male Assistant in High School.*
OAPHA B. KIMES, *1st Fem. Assistant in High School.*
ADALAIDE S. RICHARDS, *2d Fem. Assistant in High School.*
JOHN RYAN, *Principal S. W. Ward Male Grammar School.*
EMILY MANLEY, *Principal S. W. Ward Fem. Gram. School.*
H. U. HAMLIN, *Principal N. E. Ward Male Grammar School.*
AMANDA P. BRENHALTS, *Prin. N. E. Ward Fem. Gr. Sch.*
E. A. STAHLE, *Principal S. E. Ward Fem. Grammar School.*
S. L. DAVIS, *Principal N. W. Ward Fem. Grammar School.*
J. C. CLIMERSON, *Principal N. W. Ward Grammar School.*

From J. R. LOOMIS, LL. D., *President of Lewisburg University, Pa.*

I have given considerable time to an examination of Greenleaf's books, which you left with me, and am satisfied that they are a valuable series of works. They seem to me to have the merit of good arrangement and exactness of statement, which are essential points in a good text-book, and to be neither too comprehensive for the purpose which they are intended to answer, nor so condensed as to prevent a full amount of examples in illustrating the principles which they teach.

J. R. LOOMIS.

LEWISBURG, PA., Jan. 7, 1860.

From the TEACHERS OF THE PUBLIC SCHOOLS *of Mount Holly, N. J.*

MOUNT HOLLY, Feb. 3, 1860.

Greenleaf's Arithmetics were introduced into the Mount Holly Public Schools about fifteen months since, and the result of their use has been very satisfactory.

The *New Primary* is just the book for beginners; the *Intellectual* Arithmetic is admirably calculated to train the mind of the student for the application of the science to the business transactions of life; and the *National* and *Common School* Arithmetics are, in our judgment, unsurpassed.

J. P. BURNETT, JR. — R. A. STILLWELL,
A. L. BUCKLEY, — M. L. POTTS,
M. F. CARR, — R. R. SUTTER,
L. C. RHEES, — M. J. ATKINSON.

From MISS N. M. STANLEY, *Vice Principal of St. Mary's Hall, Burlington, N. J.*

For several years I have used Greenleaf's National Arithmetic, and have found it an excellent text-book. The *simplicity of the rules*, the *diversity of the examples*, and the *interesting matter found in the notes*, all recommend it to the careful teacher. The brief history of numerical science, in the introduction, is of itself a guaranty *that the book*, with a *living* teacher, will not give "gross and vulgar views;" but *furnish* "the path which leads to the knowledge of truth and reality."

N. M. STANLEY.

ST. MARY'S HALL, Jan. 8, 1860.